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# Tax Optimization under Tax Evasion:
# the "Principal-Agent Approach"

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# Tax Optimization under Tax Evasion: the “Principal-Agent Approach”

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**Abstract.**

The present paper considers a problem of optimal taxation for a group of individuals with random, independent and identically distributed incomes. An agent’s income is her private information and can only be verified through a costly audit. The purpose is to characterize the optimal tax schedule and auditing strategy that maximize net tax revenue under participation constraints related to the interests of the taxpayers. The authors determine the optimal evasion proof strategy, depending on the penalty constraint. They show that under certain conditions tax evasion may increase net tax revenue, but typically the optimal government strategy is evasion proof.

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## 1. Introduction

Informational asymmetries generate important constraints on the choice of tax policy. In much of the recent literature on optimal income taxation (Reinganum and Wilde (1985), Border and Sobel (1987), Chander and Wilde (1998) and others) individual incomes are treated as exogenous. Informational constraints arise since an individual's income cannot be directly observed. It can only be verified through a costly audit. In this setting a government strategy must include, besides tax rates, an audit strategy and a scheme of penalties for misreporting. This opens up interesting questions of interaction between the optimal tax rates, audit strategy, participation and penalty constraints.

Early optimal income tax results (see Atkinson, Stiglitz, 1980) primarily relate to the case where all taxpayers are risk-neutral, their participation constraints concern expected after tax incomes, and the government knows the type of every agent, in particular, her probabilistic distribution of income. Then the informational asymmetry is not essential: according to the theorem, the government can reach the first best result by means of a type-specific lump-sum tax, and audit is unnecessary.

However, in practice taxpayers cannot be completely risk-neutral under arbitrary tax policy. For instance, for a firm there typically exists a threshold income (dependent on its type) such that, if after-tax income falls below this value, then the firm cannot obtain access to credit and becomes bankrupt. Pervious models of optimal taxation under tax evasion (Chander and Wilde (1998), Mookherjee and Png (1989) take this condition into account in the form of a participation constraint. So, income after paying taxes and possibly fines is required to be non-negative in all circumstances, as if the firm became infinitely risk averse for outcomes below the threshold. In this case, the optimal tax in general depends on a taxpayer's income, and the determination of the optimal tax policy becomes a non-trivial task.

It should be noted that most of the literature on the optimal tax enforcement either restricts attention to linear tax schedules or considers fixed taxes and penalties (see Cremer, Marchand and Pestieau, 1990, Sanchez and Sobel, 1993).

Mookherjee and Png (1989) study the tax enforcement problem in a contract theory-type setting. They consider risk averse taxpayers and include a moral hazard problem. Their model permits arbitrary tax and penalty schedules, subject to participation constraints. Their results show that such an approach has some disadvantages: the optimal penalty schedule they find is to fine a tax evader an amount equal to the entire income, irrespective of the amount of income that was concealed. Such a draconian rule is unavailable in practice. Moreover, as Chander and Wilde (1998, p. 177) mention , it is unrealistic to assume that penalties are a choice variable of the tax authorities. Actual penalties may be constrained, for instance, by the common ethical norm of letting the punishment fit the crime.

Chander and Wilde (henceforth CW) consider the tax optimization problem for risk neutral taxpayers with an exogenous income distribution. Proceeding from the previous argument, they consider another type of penalty constraint: payment after audit is equal to the hidden income. They introduce the notion of an efficient scheme including tax, penalty and auditing probability functions such that any other scheme does not allow to increase the expected payment of any taxpayer without increasing probabilities of auditing for some reported income. They show that it is possible to restrict attention to evasion proof schemes and establish their general properties: the tax function is non-decreasing with a non-increasing average tax rate; audit probabilities are determined by the marginal payment rates and are non-increasing.

Let us note that the tax optimization problem was studied under penalty constraints that are not used in practice. Typically, identified tax evaders repay the detected unpaid tax and also pay some penalty. In several countries this penalty is proportional to the unpaid tax, in some others (including Russia) it is proportional to hidden income. As far as we know, there are no prior results establishing the optimality of evasion proof contracts given these constraints. Moreover, the previous literature establishes general properties of the optimal tax schedules, but does not provide a tool for their determination.

The present paper will study the tax optimization problem under various penalty constraints and find the optimal tax schedules depending on the characteristics of taxpayers. We will find under what conditions the optimal government strategy is evasion proof, and alternatively, possible reasons for the government to countenance some tax evasion.

Our basic model includes the government and a group of taxpayers with identical but independent random distributions of incomes. The government (the principal) knows the distribution, but does not know who has what income, while each taxpayer (the agent) knows her own income. Agents make a report of their income to the principal, while the principal may audit an agent in order to verify her income. An audit always determines the taxpayer's actual income, and each audit involves a fixed cost. The government collects taxes through the following mechanism. It sets a tax schedule (or schedule of pre-audit payments) dependent on the reported income, and a penalty schedule (or post-audit payment schedule) dependent on actual and reported incomes. The government also chooses the probability of inspection, dependent on reported income.

Our purpose is to study the tax optimization problem in this principal-agent framework, that is, to find the government strategy that maximizes expected net tax revenue (net of auditing costs) under optimal behavior of agents and participation constraints related to taxpayer income. We assume that every taxpayer is risk-neutral and aims to maximize her expected after tax income. We consider two types of participation constraints, one based on expected after-tax and penalty income, and the second based on realized after tax and penalty income:

(1) The expected income of an agent under her optimal behavior should exceed a fixed level, called her "alternative income". An otherwise unconstrained expected revenue maximizing government would find it desirable to leave a taxpayer only with her alternative income.

(2) Since the income of every agent is a random variable, we require that under optimal behavior and the worst random outcome, the actual (not expected) income of an agent should exceed a minimal value necessary for the "survival" of the taxpayer.

Further, we consider the following variants of permissible penalties imposed upon detected tax evaders: a) the penalty is proportional to the detected, unpaid tax; b) the "pure" penalty is proportional to hidden income; c) the penalty is bounded because of the required minimal after tax and penalty income of an agent; and, d) the payment after audit is proportional to detected hidden income.

Our main results are as follows. The optimal contract is always evasion proof under the penalty constraints c and d. Some tax evasion may be optimal, however, for a net revenue maximizing government if there are two possible levels of income, the pure penalty is proportional to the unpaid tax or hidden income (penalty variants a and b) and the proportionality coefficient is sufficiently low. Among evasion proof strategies the optimal tax is either equal to the entire income above the minimal level for small incomes and is flat for higher incomes, or it is flat for all incomes. In the former case the optimal audit strategy is a probabilistic cut-off rule (cf. Sanchez and Sobel, 1993): every reported income below some threshold is audited with a probability that makes underreporting unprofitable, and every higher report is not audited. Under penalty constraints and some assumptions on the density of the income distribution, the optimal evasion proof strategy is optimal in general, for continuous distributions.

The new contribution of our paper to the existing literature is as follows. Besides the penalty constraints c and d, studied by CW, we consider two other practically important variants and show that the optimal contract is not necessarily evasion proof. We examine how this property depends on the penalty coefficients, minimum alternative income and other parameters of the model. We obtain transparent results on the optimal government strategies, including the explicit tax schedules and audit rules.

The major assumptions that limit the generality of our analysis are standard in the literature, for example, are also used in CW. As in most of the literature, we rule out supply side effects of income taxation. We do not think that introducing supply effects would essentially change our results. As Mirrlees (1971) shows, these effects may only reinforce the regressivity of optimal tax schedules. Our optimal tax schedule, however, is already extremely regressive.

The other major assumption is that taxpayers are risk neutral. Since we focus on taxation of firms and the participation constraint prevents tax bankruptcy, this condition seems to be not very restrictive.

The paper proceeds as follows. Section 2.1 defines the basic model. Section 2.2 solves the tax optimization problem for the case of two possible levels of income and determines under what conditions tax evasion is profitable for the government. Section 2.3 determines the optimal evasion proof contract for different penalty constraints. Section 3 concludes with several remarks and policy implications related to the Russian economy. Technical proofs of several propositions are given in Appendix.

## 2. A Basic Model.

### 2.1. Model specification. The first best solution.

We consider an interaction between the government and a group of taxpayers. The income $I$ of each taxpayer is an independent random value with distribution function $G(I)$ concentrated in the interval $[I_L, I_H]$. The income is a private information of a taxpayer. A government strategy $s_G$, or a simple contract, includes three components: a non-decreasing tax function $T(I_r)$ where $I_r(I) \in [I_L, I]$ is a reported income, an audit probability $p(I_r) \in [0,1]$ and a penalty function $F(I, I_r)$ that determines the additional payment of the agent depending on her actual and reported incomes. In this version of the model, an audit always reveals the true income and its cost is fixed.

Under a given government strategy each taxpayer aims to maximize her expected income. So, depending on the actual income, her report is

$$I_r(I, s_G) \to \min_{I_r}\{T(I_r) + p(I_r)F(I, I_r)\} \overset{def}{=} T_{eff}(I, s_G). \qquad (2.1.1)$$

The value on the right hand is the effective tax, or the expected total payment from an agent with income $I$ under strategy $s_G$.

The problem of the government is to maximize expected tax revenue net of auditing costs[1]:

$$\int (T_{eff}(I,s_G) - cp(I_r(I,s_G)))dG(I) \to \max_{s_G} \tag{2.1.2}$$

under the following participation constraints:

$$\int (I - T_{eff}(I,s_G))dG(I) \geq I_{alt}, \tag{2.1.3}$$

that is, the expected income of an agent under her optimal behavior should exceed the alternative income;

$$T(I_r(I)) + F(I,I_r(I)) \leq I - I_{min} \text{ if } p(I_r(I)) > 0, \tag{2.1.4a}$$

$$\text{and } T(I_r(I)) \leq I - I_{min} \text{ if } p(I_r(I)) < 1, \tag{2.1.4b}$$

that is, under the optimal behavior and the worst random outcome, the income should exceed the value $I_{\min}$ which is necessary for "survival" of a taxpayer. In practice $I_{\min}$ may be a threshold value such that if the rest on the firm's account falls below this value, then the firm cannot get a credit and has to stop operating.

The value $I_{alt}$ is a desirable level of a taxpayer's expected income in our model. Another interpretation is that this is the expected income of a firm if it moves her activity to another country or untaxable field.

We consider several variants of penalty constraints:

a) $F(I,I_r) = (1+\delta_a)(T(I) - T(I_r))$ (penalty is proportional to detected unpaid tax);

b) $F(I,I_r) = T(I) - T(I_r) + \delta_b(I - I_r)$ (pure penalty is proportional to hidden income);

c) $0 \leq F(I,I_r) \leq I - T(I_r) - \hat{I}$ for any $I, I_r$ (penalty is bounded because of the given minimal income of an agent $\hat{I}(\leq I_{\min})$ under non-optimal behavior of a taxpayer);

d) $F(I,I_r) = \delta_d(I - I_r)$ (the payment after audit is proportional to detected hidden income.

Actually the government can choose the penalty only under condition c. The first inequality in this constraint means that there are no premiums neither for truthtelling nor for lie. This condition rules out the possibility of making enforcement costs arbitrary small by

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[1] In general tax revenue maximization is not the main purpose of the government. Another typical setting is that the government aims to maximize a social welfare function dependent on after tax income of taxpayer and net tax revenue (cf. Atkinson, Stiglitz, 1980). Then the optimal strategy coincides with solution of our model under a certain value of the alternative income.

offering the agents large rewards with small probabilities and thus inducing them to use strategies preferable for the government[2].

The former two variants of penalty constraints correspond to the actual legislation in different countries while the latter two are studied by CW. Their results (see Section 1 and Lemma 3 on p.177) imply the following theorem.

*Theorem 2.1.1.* Under penalty constraints c or d, there exists an evasion proof optimal government strategy for the problem (2.1.2 – 2.1.4), that is, such solution $s_G$ that $I_r(I, s_G) = I$ for any $I$. Moreover, the optimal penalty is always the maximal one in the case c: $F(I, I_r) = I - T(I_r) - \hat{I}$ for any $I \neq I_r$.

*Note.* CW consider only the case $\delta_d = 1$ for the variant d, but their proof stays true for any $\delta_d > 0$.

Section 2.2 below proves a similar result for the more general model of audit. The next section shows that the theorem is not true and tax evasion may be optimal for the government under penalty constraints a or b.

Two values play the crucial role in the subsequent analysis of the model.

$T_{LM} \overset{def}{=} I_L - I_{min}$ is a maximal possible tax on the low income under participation constraint (2.1.4). This value characterizes stability of a taxpayer under the worst state of nature.

$\Delta EI \overset{def}{=} \int I dG(I) - I_{alt}$ is an expected surplus profit of an agent before tax with respect to her alternative income. For a firm this value characterizes profitability of production.

If we exclude individual incentive constraint (2.1.1), assume that taxpayers do not evade and set $p(I_r) \equiv 0$ then we obtain the problem for the first best solution:

$$R(T(.)) = \int T(I) dG(I) \rightarrow \max_{T(I)} \qquad (2.1.5)$$

under constraints

$$\int T(I) dG(I) \leq \Delta EI \qquad (2.1.6)$$

$$T(I) \leq I - I_{\min}, \quad I \in [I_L, I_H] \qquad (2.1.7)$$

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[2] Tirole (1992) describes the corresponding contract in the general principal-agent setting. CW do not require the specified inequality but assume only that there are no large premiums for truthtelling. However, it is easy to see that large premiums for small lie may make the same effect. Moreover, we do not consider even limited premiums since they create strong incentives for collusion between the tax service and taxpayers.

Obviously, this solution does not depend on the penalty constraint. Consider the following expression for the tax function:

$$T(I) = T(I_L) + \Delta T(I),$$

where $T(I_L)$ is a lump sum component paid by each taxpayer and $\Delta T(I) \geq 0$ is an additional tax dependent on the income. Proceeding from (2.1.7), $T(I_L) \leq T_{LM}$.

*Proposition 2.1.2.* The first best net tax revenue is equal to $\Delta EI$. If $T_{LM} \geq \Delta EI$ then the optimal government strategy for the original problem (2.1.2-2.1.4) corresponds to the first best solution: $T(I) \equiv \Delta EI$, $p(I) \equiv 0$.

*Proof.* According to constraint (2.1.7), $R(I) \leq \Delta EI$. If $T_{LM} \geq \Delta EI$ then the given strategy provides revenue $\Delta EI$ and meets all constraints. Now consider the case $T_{LM} < \Delta EI$. Let us find the first best solution. Consider a function $R(\bar{I}) = \int_{I_L}^{\bar{I}} (I - I_{\min}) dG(I) + (\bar{I} - I_{\min})(1 - G(\bar{I}))$ which determines the revenue for the tax schedule $T(I) = \begin{cases} I - I_{\min}, & \text{if } I \leq \bar{I} \\ \bar{I} - I_{\min}, & \text{if } I > \bar{I} \end{cases}$. Note that $R(\bar{I})$ is continuos in $\bar{I}$ and $R(I_L) = T_{LM} < \Delta EI, R(I_H) = \int I dG(I) - I_{min} > \Delta EI$. Hence, there exists a value $\bar{I}$ such that $R(\bar{I}) = \Delta EI$.

Thus, whenever $T_{LM} \geq \Delta EI$, the government can get the first best result by means of the lump sum tax $T(I) \equiv T_L = \Delta EI$. It is unnecessary to collect any other taxes and organize audits. Proceeding from the previous discussion of conditions (2.1.3), (2.1.4), the inequality shows that the expected surplus profit is less than the maximal lump-sum payoff that does not undermine activity of a firm under unfavorable conditions. The case $T_{LM} < \Delta EI$ is more sophisticated. The next section solves the problem for one type of income distributions and shows that, under certain conditions, the government can get the maximal revenue only if taxpayers evade.

## 2.2 A model with two possible levels of income.

Consider a group of taxpayers who get a high income $I_H$ with probability $q$ and low income $I_L$ with probability $1 - q$. A government strategy includes taxes $T_L$ and $T_H$ on these incomes, a probability $p$ of auditing low income reports and a penalty $F$ on tax evasion (if this penalty is not given exogenously). Denote $\Delta I \overset{def}{=} I_H - I_L, \quad \Delta T \overset{def}{=} T_H - T_L \geq 0$. A taxpayer's strategy is her report $I_r \in \{I_L, I_H\}$ that she sends under the high income. Since a taxpayer maximizes her expected income,

$$I_r = I_L \text{ if } pF < \Delta T, \text{ otherwise } I_r = I_H. \tag{2.2.1}$$

The government aims to maximize expected net tax revenue, so the formal problem is to find

$$R \to \max_{(T_L, \Delta T, F, p)}, \tag{2.2.2}$$

where $R = T_L + q\Delta T - p(1-q)c$ if $pF \geq \Delta T$,

otherwise $R = T_L + qpF - pc$.

Participation constraints take the form

$$T_L + q\min(\Delta T, pF) \leq \Delta EI = I_L + q\Delta I - I_{alt}, \tag{2.2.3}$$

$$T_L \leq T_{LM} = I_L - I_{\min}; \; I_H - T_H \geq I_{\min} \text{ if } I_r = I_H, \tag{2.2.4}$$

$$\text{otherwise } I_H - T_L - F \geq I_{\min}.$$

The four variants of penalty constraints look in this model like

a) $F = (1+\delta_a)\Delta T;$ b) $F = \Delta T + \delta_b \Delta I;$

c) $I_L + \Delta I - T_L - \max(\Delta T, F) \geq \hat{I}$; d) $F = \delta_d \Delta I.$

If $T_{LM} \geq \Delta EI$, that is equivalent to $I_{alt} - I_{\min} \geq q\Delta I$, then, according to proposition 2.1.2, optimal net revenue $R^* = \Delta EI$ is the same for problem (2.1.1)-(2.1.4) with any penalty constraint a-d and coincides with the first best solution. The optimal government strategy is $\Delta T = 0$, $T_L = \Delta EI, \quad p = 0$.

If $T_{LM} < \Delta EI$ then the first best revenue value is the same but requires a combination of the both kinds of taxes. Let us find solutions of tax optimization problem (2.2.1)-(2.2.4) for different penalty constraints.

a) $F_a = (1+\delta_a)\Delta T$. Note that taxpayers evade in this case iff $\Delta T > 0, \; p < p_a^* \overset{def}{=} 1/(1+\delta_a)$. The proposition below shows how the optimal government strategy depends on parameters of the model.

*Proposition 2.2.1*. First, consider the case where

$$c < q\Delta I \tag{2.2.5}$$

Then, for any fixed $c, q, \Delta I$ and $\delta_a$, there exist three variants of the optimal government strategy dependent on the difference $I_{alt} - I_{\min}$. If

$$q\Delta I \le I_{alt} - I_{\min} \tag{2.2.6}$$

then the optimal strategy is to set lump-sum tax $T_L = \Delta EI (= I_L + q\Delta I - I_{alt})$, $\Delta T = 0, \quad p = 0.$

In the area

$$q\Delta I (1 - \frac{1-q}{1+\delta_a}) < I_{alt} - I_{\min} < q\Delta I \tag{2.2.7}$$

the optimal strategy includes: $T_L = T_{LM}$, the additional tax $\Delta T = \Delta I p_a^*$ (hence, $F = \Delta I$), the audit probability $p = \frac{\Delta EI - T_{LM}}{q\Delta I} \ (= 1 - \frac{I_{alt} - I_{\min}}{q\Delta I}) < p_a^*$, so tax evasion is optimal for the government. The gross revenue $R_g$ is $\Delta EI$ (that is maximal under (2.2.3)), and the net revenue $R = \Delta EI - pc$.

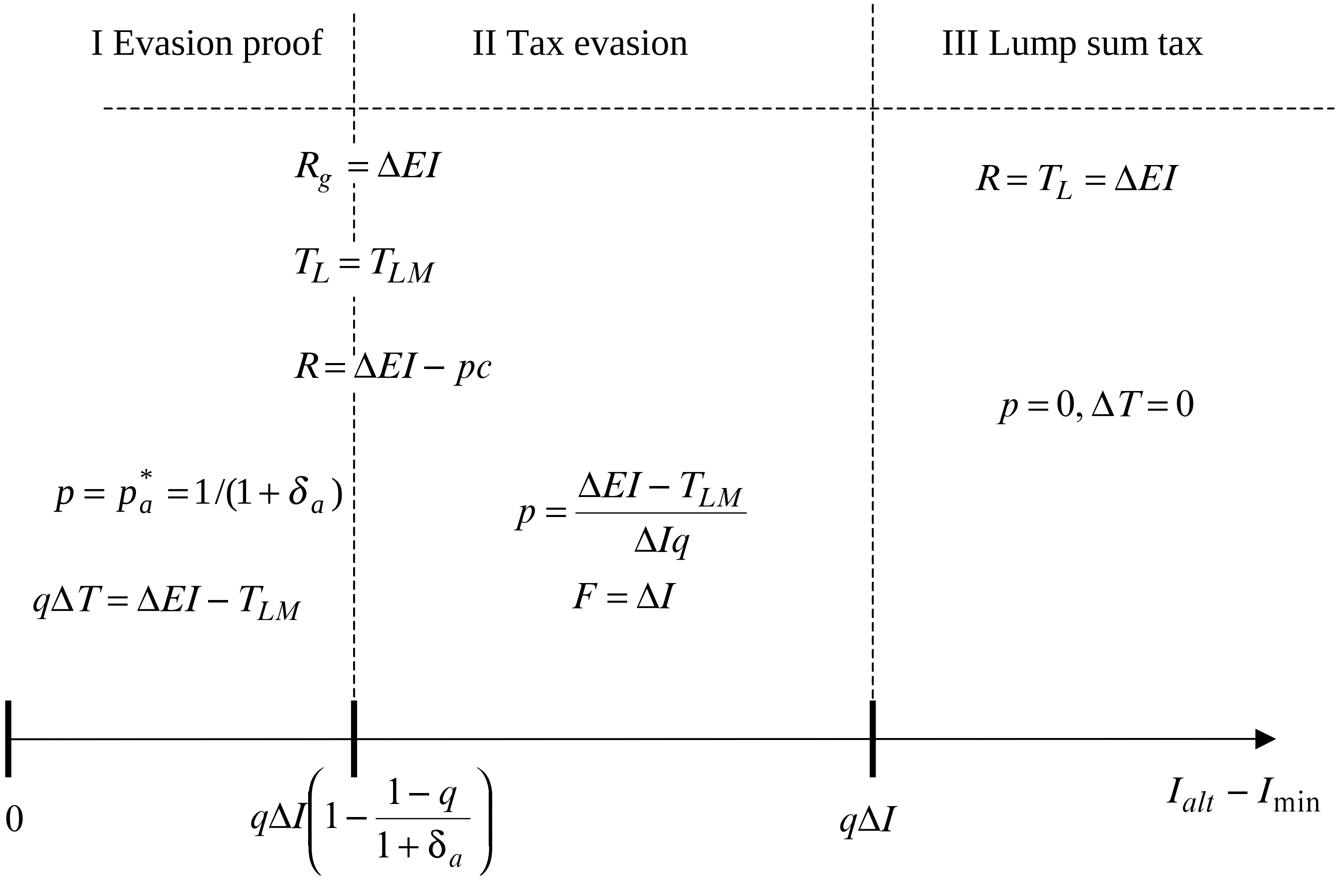
In the rest area

$$0 < I_{alt} - I_{\min} \le q\Delta I (1 - \frac{1-q}{1+\delta_a}) \tag{2.2.8}$$

the optimal strategy is $T_L = T_{LM}, \quad p = p_a^*, \Delta T = (\Delta EI - T_{LM})/q$. Honest behavior of taxpayers is optimal, and the gross revenue $R_g$ is again $\Delta EI$ while the net revenue $R = \Delta EI - (1-q) p_a^* c$.

Figure 1 summarizes our results on the optimal government strategy under condition (2.2.5).

**Figure 1. The optimal tax enforcement strategy under penalty constraint $F_a = (1+\delta_a)\Delta T$ and condition $c < q\Delta I$.**



If $c(1-q)p_a^* < q\Delta I \le c$ then the optimal government strategy corresponds to Figure 2. In the area

$$q\Delta I - c(1-q)p_a^* \le I_{alt} - I_{\min} < q\Delta I$$

the optimal strategy is $(T_L = T_{LM},\ p = 0,\ \Delta T = 0)$. The gross and net revenue under this strategy is $R = T_{LM} < \Delta EI$.

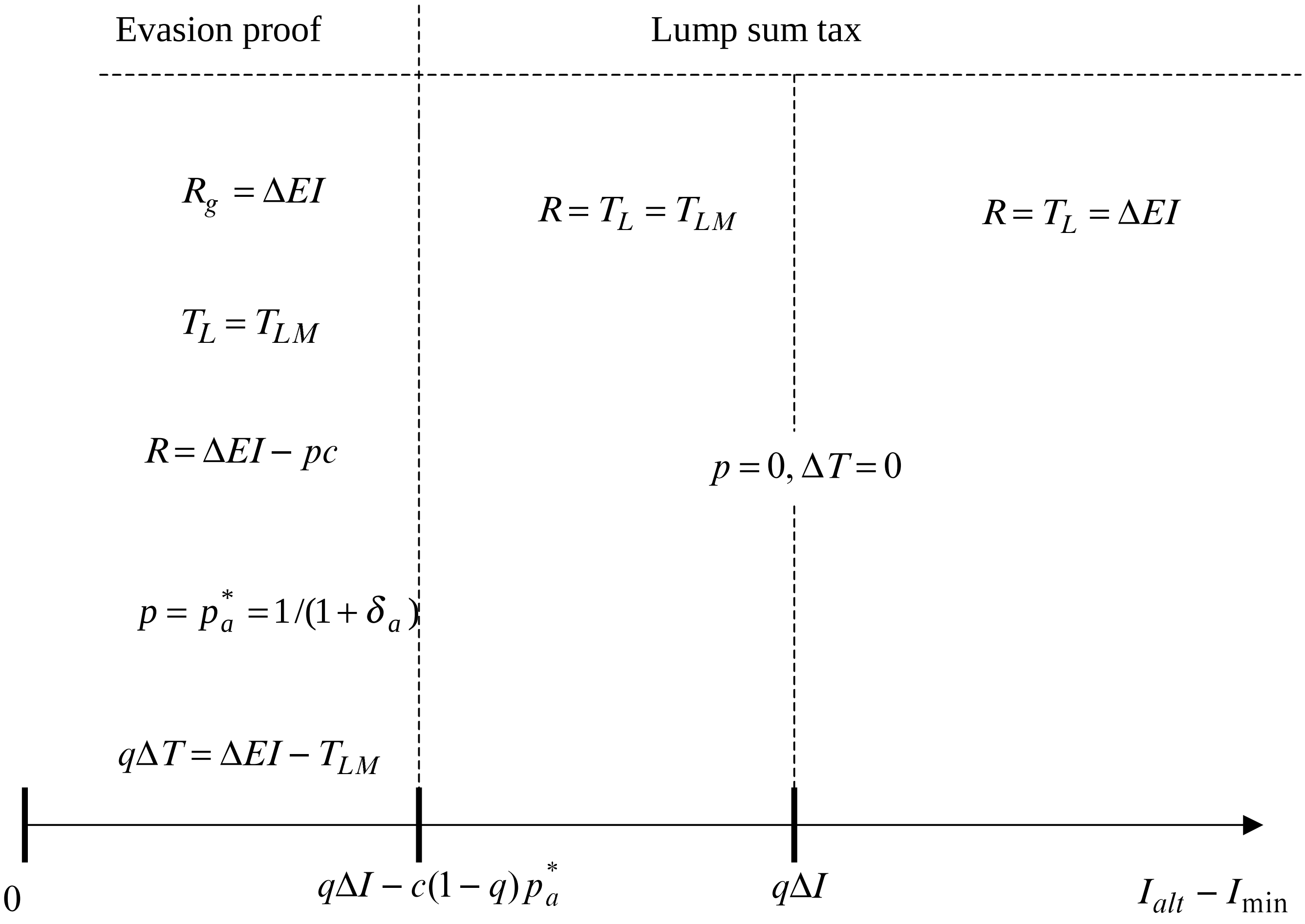
In the area

$$0 \le I_{alt} - I_{\min} < q\Delta I - c(1-q)p_a^*$$

the optimal strategy is ( $T_L = T_{LM},\ \ p = p_a^*, \Delta T = (\Delta EI - T_{LM})/q$ ), as in the area (2.2.8) under (2.2.5).

Finally, under $q\Delta I \le c(1-q)p_a^*$ lump sum tax always provides the maximal revenue. If $I_{alt} - I_{\min} < q\Delta I$ then the optimal strategy is $(T_L = T_{LM},\ p = 0,\ \Delta T = 0)$.

**Figure 2. The optimal tax enforcement strategy under penalty constraint** a) **and condition** $c(1-q)p_a^* < q\Delta I \le c$ **.**



Let us discuss this result.

Return to the case $c < q\Delta I$. Since $\Delta I$ characterizes variance of the income distribution (under a fixed $q$), we may say that lump-sum taxation is optimal when the variance is sufficiently small with respect to the difference between $I_{alt}$ and $I_{\min}$. If the variance is relatively large then the optimal strategy is evasion proof, and in the intermediate area tax evasion is optimal for the government. Another interpretation relates to the values $\Delta EI$ and $T_{LM}$.

Note that inequalities (2.2.6-2.2.8) are equivalent to

$$0 \le T_{LM} - \Delta EI;$$

$$0 < \Delta EI - T_{LM} < q\Delta I \frac{1-q}{1+\delta_a};$$

$$q\Delta I\left(\frac{1-q}{1+\delta_a}\right)\le \Delta EI - T_{LM} < q\Delta I$$

respectively. Since the difference $\Delta EI - T_{LM}$ characterizes profitability of production versus its stability, we may conclude that lump-sum taxation is optimal when profitability is relatively low (area (2.2.6)), the monotone tax and the audit strategy enforcing honest reporting are optimal where profitability is relatively high (2.2.8), and the monotone tax with tax evasion is optimal in the intermediate area (2.2.7). Figure 2 shows that if the cost of audit is relatively high $(c \ge q\Delta I)$ then lump-sum taxation is more profitable than permission of tax evasion in the intermediate area. In other respects the picture is similar to the previous case.

Intuition for this result is as follows.

According to (2.2.1), the whole set of government strategies divides into three subsets: area I and $pF \ge \Delta T > 0$ and honest reporting is optimal for taxpayers, area II, where $\Delta T \ge pF > 0$ where it is optimal to evade from tax, and area III where $\Delta T = 0$. In the latter area the optimal auditing rule is obviously $p = 0$ and the maximal revenue under $T_{LM} < \Delta EI$ is $T_{LM}$ irrespective of the penalty constraint.

In the areas I and II the optimal strategy includes the maximal possible lump sum tax $T_L = T_{LM}$ (its collection does not require any audit costs). In the area II the revenue is collected in the form of penalties. The maximal gross revenue is $\Delta EI$. In order to get it, we set $pF = (\Delta EI - T_{LM})/q$. Under this condition, the penalty should be as large as possible, since we aim to minimize audit costs. Because of participation constraint (2.2.4), $F_{max} = \Delta I$. Condition (2.2.5) shows that collection of penalties is profitable under such fine.

In the area I the revenue comes from taxes. By similar argument, $\Delta T = (\Delta EI - T_{LM})/q$, and the minimal audit probability that makes honest reporting optimal is $p_a^*$. By comparison of $p_a^*$ with the optimal audit probability in the area II, we determine the broad between areas II and III. In other cases reasoning is similar. See Appendix for the detailed proof.

Now consider constraint b) $F = \Delta T + \delta_b \Delta I$.

*Proposition 2.2.2.* Let $c < q\Delta I, \quad q + \delta_b < 1$. Then in the area

$$q\Delta I(q+\delta_b) < I_{alt} - I_{\min} < q\Delta I$$

the optimal strategy includes: $T_L = T_{LM}$, the additional tax $\Delta T = \Delta I(1-\delta_b)$ (hence, $F = \Delta I$), the audit probability $p = \dfrac{\Delta EI - T_{LM}}{q\Delta I}$ $(=1-\dfrac{I_{alt} - I_{\min}}{q\Delta I}) < \Delta T / F$, so tax evasion is optimal for the government. The gross revenue $R_g$ is $\Delta EI$, and the net revenue $R = \Delta EI - pc$.

In the area

$$0 < I_{alt} - I_{\min} \le q\Delta I(q + \delta_b)$$

the optimal strategy is

$$T_L = T_{LM},\ \Delta T = (\Delta EI - T_{LM})/q,\ p_b^* = \Delta T/(\Delta T + \delta_b \Delta I). \qquad (*)$$

Honest behavior of taxpayers is optimal, and the gross revenue $R_g$ is again $\Delta EI$ while the net revenue $R = \Delta EI - (1-q)p_b^* c$. For any other values of parameters either strategy (*) or the lump sum tax $(T_L = T_{LM}, p = 0, \Delta T = 0)$ are optimal.

Proof is given in the Appendix. Note that, in contrast to the case a, if the penalty coefficient $\delta_b$ is sufficiently large $(>1-q)$ than the optimal contract is always evasion proof.

Thus, for certain parameter values, the optimal contract implies tax evasion under constraints a or b. This is in contrast to the cases c and d, where an optimal contract which is evasion proof always exists according to the Theorem 2.1.1.

Let us specify the optimal strategies. Consider the problem under c) $F \le I_H - \widehat{I} - T_L$.

*Proposition 2.2.3.* In the area $\Delta EI > T_{LM}$, the optimal government strategy is ($T_L = T_{LM}$, $\Delta T = (\Delta EI - T_{LM})/q$, $F = F_c \overset{def}{=} I_H - \widehat{I} - T_{LM}$, $p = p_c \overset{def}{=} \Delta T / F_c$) if $qF_c > (1-q)c$. Then the gross revenue $R_g$ is $\Delta EI$ while the net revenue $R = \Delta EI - (1-q)p_c c$. Under $qF_c \le (1-q)c$ the lump sum tax $(T_L = T_{LM}, p = 0, \Delta T = 0)$ is optimal for the government.

*Proposition 2.2.4.* Under penalty constraint d) $F_d = \delta_d \Delta I$, the optimal government strategy in the area $\Delta EI \ge T_{LM}$ is ($T_L = T_{LM}$, $\Delta T = (\Delta EI - T_{LM})/q$, $p = p_d \overset{def}{=} \Delta T / F_d$) if $qF_d > (1-q)c$. Then the gross revenue $R_g$ is $\Delta EI$ while the net revenue $R = \Delta EI - (1-q)p_d c$. Otherwise, the lump sum tax $(T_L = T_{LM},\ p=0, \Delta T = 0)$ is optimal for the government.

See Appendix for the proofs.

Now, let us compare the penalty constraints from the point of view of the government.

Under any constraint a, b or d, the maximal revenue increases (or, at least does not decrease) in the penalty coefficient $\delta_a, \delta_b$ or $\delta_d$ respectively and tends to the first best value $\Delta EI$ as the coefficient tends to infinity. In the case c the same proposition hold if we set $\delta_c = -\widehat{I}$. So the reasonable way to compare the penalty constraints is to find the equivalent values of the penalty coefficients. According to Propositions 2.2.1-2.2.4, we should consider several areas of parameters of the model. Let us start with

$$I_{alt} - I_{\min} < q\Delta I,$$

$$\delta_a \geq \max\{0, \delta_a^*\} \text{ where } \delta_a^* \overset{def}{=} (I_{alt} - I_{\min} - q^2\Delta I)/(q\Delta I - I_{alt} + I_{\min}), \qquad (2.2.10)$$

$\delta_a^*$ is a solution of the equation $I_{alt} - I_{\min} = q\Delta I(1 - \dfrac{1-q}{1+\delta})$. Note that under $I_{alt} - I_{\min} < q^2\Delta I$ such solution is negative.

Let $R_x^N$ denote the maximal net revenue in strategy area $N \in \{I, II, III\}$ under constraint $x \in \{a,b,c,d\}$. Then $R_a^I \geq R_a^{II}$ under (2.2.10) by proposition 2.2.1. The optimal strategy in the area I is

$$\Delta T = \Delta T^* \overset{def}{=} (\Delta EI - T_{LM})/q, \quad T = T_{LM}, \quad p = 1/(1+\delta_a), \qquad (2.2.11)$$

the penalty for evasion is $F = \Delta T^*(1+\delta_a)$. Let us determine $\delta_b, \widehat{I}$ and $\delta_d$ from equations:

$$F = \Delta T^*(1+\delta_a) = \Delta T^* + \Delta I\delta_b = \Delta I + I_{\min} - \widehat{I} = \Delta I\delta_d. \qquad (2.2.12)$$

Then for any penalty constraint strategy (2.2.11) is optimal in the area I. Government revenue and incomes of all agents do not depend on the penalty constraint under (2.2.11-12). Since the optimal strategy in the area III (where $\Delta T = 0$) and $R_x^{III}$ do not depend on the penalty constraint, the same proposition is true for the globally optimal strategy. Thus, relation (2.2.12) determines the equivalent penalty coefficients under (2.2.11).

Now consider $I_{alt} - I_{\min} > q^2\Delta I,\ \ \delta_a \in (0, \delta_a^*)$. Then $R_a^{II} > R_a^I$, the optimal strategy in the area II is

$\Delta T = \Delta I/(1+\delta_a)$ (so $F=\Delta I$), $T_L = T_{LM}$, $p = (\Delta EI - T_{LM})/q\Delta I$.

Note that government revenue and income distributions of taxpayers do not depend on $\delta_a$ since the agents evade from tax. Thus, all values $\delta_a \in \{0, \delta_a^*\}$ are equivalent in this sense. The only component that changes is a nominal tax rate $\Delta T$.

It is easy to check that $\Delta T^*(1+\delta_a^*) = \Delta I(1-q)$. Equations (2.2.12) determine equivalent values $\delta_b^*, \widehat{I}^*$ and $\delta_d^*$ for $\delta_a^*$:

$$\delta_b^* = \frac{\Delta T^* \delta_a^*}{\Delta I} = (I_{alt} - I_{\min} - q^2\Delta I)/q\Delta I,$$

$$\delta_d^* = \frac{\Delta T^*}{\Delta I}(1+\delta_a^*) = 1-q,$$

$$\widehat{I}^* - I_{\min} = q\Delta I.$$

But $\widehat{I} \le I_{\min}$, so the equivalent value $\widehat{I}^*$ exists only for $\delta_a \ge \overline{\delta}_a \overset{def}{=} \Delta I/\Delta T^* - 1$. Note that $F(\overline{\delta}_a) = \Delta I$.

Under constraint b, $R_b^{II} > R_b^I$ if $0 < \delta_b < \delta_b^*$ and $I_{alt} - I_{\min} > q^2\Delta I$. The optimal strategy in II is the same, except for $\Delta T = \Delta I(1-\delta_b)$ (such that $F = \Delta I$), and brings the same incomes, gross and net revenues. All $\delta_b \in (0, \delta_b^*)$ and equivalent as well as $\delta_a \in (0, \delta_a^*)$.

Under constraint d, $R_d^I \ge R_d^{II}$, and for any $\delta_d < \delta_d^*$ $R_d^I(\delta_d) < R_d^I(\delta_d^*) = R_a^I(\delta_a^*) = R_a^{II}(\delta_a)$ for $\delta_a \in (0, \delta_a^*)$. Thus, in contrast to the cases a and b, net revenue decreases together with $\delta_d$ until it reaches $T_{LM} = R_d^{III}$.

### 2.3 The optimal evasion proof strategies.

Now we return to the general model with income distribution $G(I)$. According to the Theorem 2.1.1, the optimal contract is always evasion proof under penalty constraints c, d. Proceeding from Propositions 2.2.1-2.2.2, we may assume that the same is true under constraints a or b if the penalty coefficients are sufficiently large. This section finds the optimal evasion proof strategy for the problem (2.1.2-2.1.4) under the following penalty constraint that generalizes a, b and d:

$$F = k\Delta T + l\Delta I, \text{ where } k,l \ge 0,\ \Delta I = I - I_r;\ \Delta T = T(I) - T(I_r); \qquad (2.3.1)$$

note that a) corresponds to $l = 0$, b – to $k = 1$, d – to $k = 0$. Moreover, we show that this strategy is optimal in general under certain assumptions.

For any tax schedule $T$ let $\bar{I}$ - denote such minimal income that $T(I) = T(\bar{I})$ for any $I > \bar{I}$, that is, the tax is flat for greater incomes. Then strategy $s_G = (T, p)$ is evasion proof under (2.3.1) if and only if

$$p(I_r) \geq \Delta T / F \text{ for any } I > I_r, \quad I_r < \bar{I}. \tag{2.3.2}$$

So the tax optimization problem for evasion proof strategies is as follows:

$$(T(.), p(.)) \rightarrow \max(R_g(T(.)) - C(p(.))), \tag{2.3.3}$$

where $R_g(T(.)) = \int T(I)dG(I)$ is gross revenue, $C(p(.)) = c\int p(I)dG(I)$ is a total audit cost, the government strategy meets condition (2.3.2) and participation constraints

$$R_g(T(.)) \leq \Delta EI = \int IdG(I) - I_{alt}, \tag{2.3.4}$$

$$0 \leq T(I) \leq I - I_{\min}. \tag{2.3.5}$$

The following propositions characterize the optimal tax schedule and audit rule for the problem (2.3.1 – 2.3.5).

*Proposition 2.3.1.* The optimal tax schedule $T$ is concave, that is, $T(\lambda I_1 + (1-\lambda)I_2) \geq \lambda T(I_1) + (1-\lambda)T(I_2)$ for any $I_1 < I_2$, $\lambda \in [0,1]$. For any concave tax schedule, the optimal audit probability is

$$p(I_r, T) = (k + l / T'_+(I_r))^{-1}, \tag{2.3.6}$$

where $T'_+(I_r)$ is the marginal tax rate for income $I_r$.

*Proof.* Note that gross revenue $R_g$ does not depend on the audit rule in this case. For any tax schedule $T$, the optimal evasion proof audit rule that minimizes the audit cost is

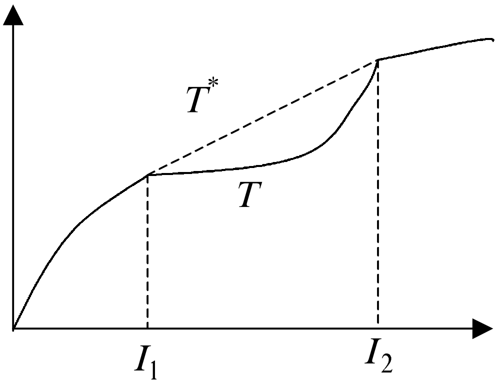
$$p(I) = \sup_{\Delta I > 0}(\Delta T / (k\Delta T + l\Delta I)) \text{ for any } I < \bar{I},\ p(I) = 0 \text{ for any } I \geq \bar{I}. \tag{2.3.7}$$

So the optimal probability of audit of any report $I_r$ is equal to minimum that makes reporting $I_r$ unprofitable for any $I > I_r$.

Assume from the contrary that $T$ is not concave. Then its linear approximation in some interval lies above $T$ (see figure 3). Let us change $T$ for its linear approximation in this interval and denote the new tax schedule by $T^*$. Then gross revenue will increase. Proceeding from (2.3.7), the optimal probability will not increase for any $I_r$ because

$\sup\frac{\Delta T}{\Delta I}=\sup\frac{\Delta T^*}{\Delta I}$ for any $I \le I_1$ or $I \ge I_2$, and $\sup\frac{\Delta T}{\Delta I}\ge\sup\frac{\Delta T^*}{\Delta I}$ for any $I\in(I_1,I_2)$. The only problem is that $R_g(T^*)$ may not meet constraint (2.3.4).

**Figure 3.**



For any schedule $T$ and tax level $Y\in[0,T(I_H)]$, let $T_Y$ denote the following tax schedule: $T_Y(I)=T(I)$ if $T(I)\le Y$, otherwise $T(I)=Y$. Note that $R_g(T_Y)$ continuously changes from $0$ to $R_g(T)$ while $Y$ changes from $0$ till $T(I_H)$. The optimal audit probability $p(I_r,T_Y)$ does not decrease in $Y$ for any $I_r$. So, if $R_g(T^*)$ exceeds $\Delta EI$ then we can choose $Y<T(I_H)$ such that $R_g(T_Y)=\Delta EI$, and reduce audit costs. Hence $T$ is not optimal. Finally, for any concave tax schedule, the ratio $\Delta T/\Delta I$ does not increase in $\Delta I$, so we obtain expression (2.3.6) from (2.3.7) as $\Delta I$ tends to $0$.

*Note.* CW obtained similar results under the penalty constraint $F=\Delta I$. This proposition also holds if the penalty is progressive in $\Delta I$, that is, $k$ depends on $\Delta I$ and increases in it.

Let there exist a density $\rho(I)=dG(I)/dI$ of the income distribution and $\mu(I)=(1-G(I))/\rho(I)$ denote its hazardous rate. For typical statistic distributions, such as lognormal, uniform etc., this rate decreases in $I$. The following theorem finds the optimal evasion proof government strategy for any income distribution with a decreasing hazardous rate. If $\Delta EI\le T_{LM}$ then, according to Proposition 2.1.2, the optimal strategy is lump-sum tax $T\equiv\Delta EI$. Now let $\Delta EI>T_{LM}$.

*Theorem 2.3.2.* If $\mu(I)$ decreases in $I$ then the optimal tax schedule $T$ for the problem (2.3.1-2.3.5) meets the following conditions: there exists $\bar{I}$ such that $T(I)=I-I_{\min}$ for any $I\le\bar{I}$ and $T(I)=T(\bar{I})$ for any $I>\bar{I}$. The optimal threshold $\bar{I}$

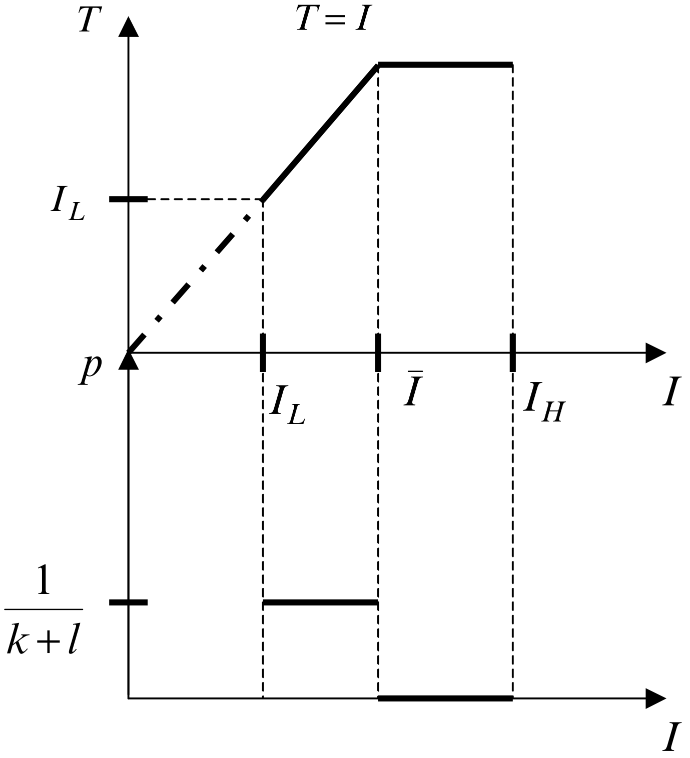
meets either conditions $R_g(T) = \Delta EI$ and $\mu(\bar{I}) > \frac{c}{k+l}$, or $R_g(T) < \Delta EI$ and $\mu(\bar{I}) = \frac{c}{k+l}$. The optimal audit rule is $p(I) \equiv 1/(k+l)$ for $I < \bar{I}$, $p(I) \equiv 0$ for $I \geq \bar{I}$.

Moreover, if the penalty is proportional to the unpaid tax (penalty constraint a)) $(k = 1 + \delta_a, l = 0)$ the specified strategy is optimal in general for initial problem (2.1.2-2.1.4).

See Appendix for the proof.

The intuition is that, under the optimal audit rule, net tax revenue monotonously depends on marginal tax rates $T_+^{'}(I)$, so the optimal rate is either the maximal $(=1)$ or the minimal $(=0)$. Since $T$ is concave by proposition 2.3.1, it corresponds to Figure 4.

**Figure 4. The optimal evasion proof strategy under $I_{\min} = 0,\ \Delta EI > T_{LM}(= I_L)$.**



Thus, the optimal tax takes the whole income above the minimum for $I < \bar{I}$ and is flat for incomes above $\bar{I}$. The optimal audit strategy is a known cut-off rule (see Sanchez, Sobel, 1993): reports below the threshold $\bar{I}$ are audited with the minimal probability that makes tax evasion unprofitable, and reports above $\bar{I}$ are not audited.

We conclude this section by comparison of different penalty constraints. Theorem 2.3.2 shows that the optimal evasion proof strategy and net revenue do not depend on the ratio of penalty coefficients $k$ and $l$. The penalty proportional to the unpaid tax is equivalent t o the penalty proportional to hidden income since the optimal tax is equal to the income above the minimal level. Expenses on audit decrease and net revenue increases in the sum $k+l$.

## 3. Concluding remarks.

In this paper we have studied the tax optimization problem with account of tax evasion for a group of taxpayers with random, independent and equally distributed incomes. Our results clarify the role of participation and penalty constraints in determination of the optimal tax enforcement strategy.

According to the known Welfare theorem, if the government has a complete information on the income distribution, and the taxpayers are risk-neutral, then it is optimal to impose a lump-sum tax, and not to organize audit. However, if there exists a constraint on the minimal income under the worst state of nature, then it may be optimal to combine a lump sum tax with other types of taxes. In this case the maximal lump sum tax is limited, on the one hand, by the value $T_{LM}$ that can undermine a firm's activity under unfortunate production conditions, on the other hand, by the value $\Delta EI$, that is the surplus expected profit before tax with respect to the desirable level of after tax profit for this group.

Our results show that it is always optimal to set the maximal lump sum tax under the mentioned constraints. If the surplus value is relatively small then introduction of any other tax does not increase net revenue. But if the profit distribution is widely dispersed and the cost of audit is relatively low then the optimal evasion proof contract has the following structure: all incomes below some threshold are taxed to the minimal income, and for all higher incomes the tax is flat. The optimal audit strategy is a known "cut-off" rule: reports below the threshold are audited with the minimal probability that makes tax evasion unprofitable and the rest reports are not audited. In some sense the optimal contract does not depend on the penalty rule: it does not matter if the penalty is proportional to evaded tax or hidden income, or includes the both components. Since the tax is equal to the whole income above the minimal level, only the sum of penalty coefficients matters.

In order to find the optimal threshold income, we may start with the lowest level and increase it until the marginal expenses of audit exceed the marginal revenue or the expected after tax profit of the agents reaches its desirable level.

Our formal result is similar to the known propositions on optimality of regressive taxes (see Mirrlees, CW et al.). However, note that the expected income is the same for every taxpayer in our case. We just show that it is optimal to praise fortunate agents with higher incomes.

Surprisingly, this result is relevant to the practice of taxation in Russia. A typical approach is that, for any tax period, a tax service establishes normative tax levels for firms under her supervision proceeding from ex ante information on these firms and some general characteristics of the market in this period (see Metodika, 1997). Such tax level corresponds to the threshold tax in our model. Only firms that do not pay this normative tax are carefully audited and penalized. One essential difference from our model is that the tax (as well as the penalty) may have another form and go not to the government revenue, but another way (for instance, a “voluntary” donation that goes to some funds of the local administration, see Yakovlev, 2000). Note that criminal groups use a similar approach to enterprises under their control. But this is another story.

One difficulty related to this approach is how to determine desirable after tax incomes $I_{alt}$ for different groups of taxpayers. From the theoretical point of view, these values should proceed from solution of the general welfare optimization problem for the economy. In this context our paper permits to exclude the tax evasion issue from this general problem since we have determined the minimal cost of tax collection for a given desirable income $I_{alt}$.

An important issue is whether the specified contract is optimal in general, that is, the optimal tax policy of the government is always evasion proof. In contrast to conventional point of view (see Chander and Wilde (1998), Mookherjee and Png (1989) and others), we found that tax evasion may be optimal for the government if the penalty for evasion is a surcharge on unpaid tax or the fine is proportional to hidden income. This happens if there are two possible levels of income, the penalty for evasion is sufficiently soft and profitability of production (characterized by $\Delta EI$) is neither very high, nor very low, but exceeds $T_{LM}$ to some extent (recall Figure 1).

However, for typical continuous distributions of income (uniform, exponential and others), the optimal evasion proof contract is optimal in general if the penalty is proportional to evaded tax.

In this paper we put aside two practically important problems. Actually the government deals with heterogeneous taxpayers. In many cases it has rather incomplete information on their initial characteristics (that is, income distributions), and cannot propose the type-specific contract. We obtained some results for the model of such sort with nonrandom incomes in the complementary paper by Marhuenda, Vasin, Vasina, 2000. However, investigation of the model including the both kinds of informational asymmetry stays a challenging task.

Another important issue is corruption in tax administration. (A recent research on this subject is Hindriks et al, 1999.) Let us note that inclusion of a possibility of collusion between a taxpayer and her auditor in our model does not essentially change the formal results on the optimal government strategy. By introduction of premiums to auditors for revealed tax evasion the government can eliminate incentives for bribe taking and obtain the same optimal tax revenue (see Vasin and Panova, 1999). Kofman and Lawarree (1993, 1996) derive a similar result for the Principal-Supervisor-Agent model in another context. The role of giving incentives to the state officials in fighting corruption was also recognized within the economic literature (see Bardhan, 1977). The actual problem is that the condition of random interaction between taxpayers and auditors in the mentioned models is in contradiction with practice in many cases. For long-time relations between a taxpayer and an auditor the mentioned scheme of premiums is not so efficient. So, this problem needs additional investigation.

## 4. Appendix.

*Proof of Proposition 2.2.1.* Proceeding from proposition 2.1.2, it suffices to study the case $I_{alt} - I_{\min} < q\Delta I$. Let $y = pF$ denote an actual expected payment to the budget from the additional income. The tax optimization problem in the area II can be set as

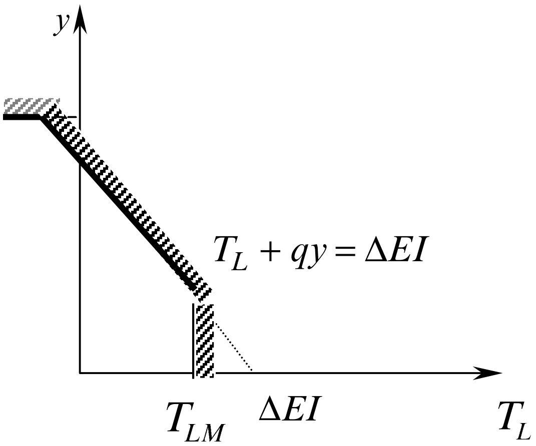
$$T_L + qy - pc \to \max \tag{4.1}$$

under $T_L + qy \le \Delta EI, \quad T_L \le T_{LM}, \quad T_L + \frac{y}{p} \le T_{LM} + \Delta I.$ Under any permissible $T_L, y > 0,$ the optimal $p$ turns the latter relation into equity. Thus, we should maximize

$$R(T_L, y) \overset{def}{=} T_L + qy - cy/(T_{LM} - T_L + \Delta I)$$

at the area represented at Figure 5.

**Figure 5.**



Condition (2.2.5) implies that $R(T_L, y)$ increases in $y$ under any permissible $T_L$. On the line $T_L + qy = \Delta EI$ the revenue is $\Delta EI - cp(y)$, where $p(y) = y/(\Delta I + T_{LM} - \Delta EI + qy)$. The audit probability increases in $y$ and reaches its minimum at $T_L = T_{LM}$, $y = \frac{(\Delta EI - T_{LM})}{q}$. Thus, the maximal revenue is $R^{II} = \Delta EI - c(\Delta EI - T_{LM})/(\Delta Iq)$, $F = \Delta I$.

In order to find the optimal government strategy it suffices to compare $R^{II}$ with the maximal revenues in the areas I and III. In the latter area $R^{III} = T_{LM}$ since $\Delta T = 0$. Thus, $R^{II} > R^{III}$ because of (2.2.5).

In the area I $R^{I} = \Delta EI - (1-q)\frac{c}{1+\delta}$ and $R^{II} > R^{I}$ iff $\frac{\Delta EI - T_{LM}}{\Delta Iq} < \frac{1-q}{1+\delta}$ that is equivalent to $I_{alt} - I_{\min} > q\Delta I(1 - \frac{1-q}{1+\delta})$. If $q\Delta I \le c$ then $R^{III} \ge R^{II}$, and $R^{III} \ge R^{I}$ iff $I_{alt} - I_{\min} \ge q\Delta I - c(1-q)p_a^{*}$.

*Proof of Proposition 2.2.2.* In the area II setting and solution of the tax optimization problem is the same as under a): $F = \Delta I$, $p^{II} = (\Delta EI - T_{LM})/q\Delta I = \frac{q\Delta I - I_{alt} + I_{\min}}{q\Delta I}$; the only difference is that $\Delta T = \Delta I(1-\delta_b)$ in this case.

In the area I $\Delta EI > T_{LM} \quad \Rightarrow \quad R^{I} = \max(T_L + q\Delta T - (1-q)cp)$

under constraints

$$p(\Delta T + \delta\Delta I) \ge \Delta T, \ T_L \le T_{LM}, \quad T_L + q\Delta T \le \Delta EI, \quad \Delta T \le \Delta I.$$

Then $p^{I} = \frac{\Delta T}{\Delta T + \delta\Delta I}$, and net revenue reaches its maximum either when

$$T_{L:} + q\Delta T = \Delta EI, \ T_L = T_{LM}, \ \Delta T = \Delta I - (I_{alt} - I_{\min})/q.$$

As well as under a), $R^{III} = T_{LM}$, and $R^{II} > R^{III}$ iff $q\Delta I > c$. On the hand, $R^{II} > R^{I}$ iff $p^{II} = (\Delta EI - T_{LM})/q\Delta I < p^{I}(1-q)$.

$$\Delta EI - T_{LM} + \delta_b q\Delta I < q\Delta I(1-q) \Leftrightarrow (q + \delta_b)q\Delta I < I_{alt} - I_{\min} < q\Delta I.$$

If $q\Delta I \le c$ then $R^{III} > R^{II}$, and $R^{III} > R^{I}$ iff

$$T_{LM} \geq \Delta EI - c(1-q)\frac{\Delta EI - T_{LM}}{\Delta EI - T_{LM} + q\delta_b \Delta I} \Leftrightarrow$$

$$\Leftrightarrow c(1+q) \geq q\Delta I(1+\delta_b) - (I_{al} - I_{\min}) \Leftrightarrow$$

$$\Leftrightarrow q\Delta I \geq I_{alt} - I_{\min} \geq q\Delta I(1+\delta_b) - c(1-q).$$

*Proof of Proposition 2.2.3* In the area I we aim to maximize $R^I = (T_L + q\Delta T - (1-q)cp)$ under constraints $F + T_L \leq I_H - \widehat{I}$, $T_L \leq T_{LM}$, $T_L + q\Delta T \leq \Delta EI$ and $pF \geq \Delta T$. Thus, the optimal $p^* = \frac{\Delta T}{F}$, $F^*(T_L) = I_H - \widehat{I} - T_L$. If $qF_c > (1-q)c$ for any $T_L \leq T_{LM}$ then $q - (1-q)c / F^*(T_L) > 0$ for any $T_L \leq T_{LM}$, $T_L^* + q\Delta T^* = \Delta EI$ and the optimal $T_L^*$ minimizes the audit probability proportional to $\frac{\Delta EI - T_L}{I_H - \widehat{I} - T_L}$. Since $I_{alt} > I_{\min} > \widehat{I}$, $\Delta EI = I_L + q\Delta I - I_{alt} < I_H - \widehat{I}$, and $T_L^* = T_{LM}$. If $qF_c \leq (1-q)c$ then $\Delta T^* = 0$, $T_L^* = T_{LM}$.

*Proof of Proposition 2.2.4* is similar to the previous one. The only difference is that $F_d^* = \delta_d \Delta I$, so $q\Delta T^* + T_L^* = \Delta EI$ if $qF_d > (1-q)c$.

*Proof of Theorem 2.3.2.* According to Proposition 2.3.1, we can express the net revenue under the optimal audit strategy as follows:

$$\int (T_+'(I)(\mu(I) - (kT_+'(I) + l)^{-1}))dG(I).$$

Since the hazardous rate and $T_+'$ decrease in $I$, the coefficient after $T_+'$ monotonously decreases in $I$. Thus, under the optimal $T$, the marginal tax rate $T_+'$ is maximal, that is equal to 1, till some $\bar{I}$, and is minimal, that is equal to 0, since this $\bar{I}$.

Let $R(\bar{I})$ (respectively $R_g(\bar{I})$) denote net (respectively gross) revenue under the tax schedule with threshold $\bar{I}$. Then

$$R(\bar{I}) = \int_{I_L}^{\bar{I}} \rho(I)(I - \frac{c}{k+l})dI + (1 - G(\bar{I}))\bar{I},$$

$$R'(\bar{I}) = \rho(\bar{I})(\bar{I} - \frac{c}{k+l}) - \rho(\bar{I})\bar{I} + 1 - G(\bar{I}).$$

If $\mu(I_L) > \frac{c}{k+l}$ then $R(\bar{I})$ is a unimodal function with a unique maximum at the point $\tilde{I}$ such that $\mu(\tilde{I}) > \frac{c}{k+l}$. The optimal threshold $I^*$ is equal to $\tilde{I}$ if $R_g(\tilde{I}) \le \Delta EI$, otherwise $R_g(I^*) = \Delta EI$. Under the opposite inequality ($\mu(I_L) \le \frac{c}{k+l}$), lump sum tax $T \equiv T_{LM}$ is optimal.

Now consider the case where $l = 0$, $k = 1 + \delta_a$. In this case condition (2.1.4) takes the form

$$T(I) + \delta_a(T(I) - T(I_r(I))) \le I - I_{\min} \text{ if } q(I) > 0 \qquad \text{(a)}$$

$$\text{and } T(I_r(I)) \le I - I_{\min} \text{ if } q(I) < 1. \qquad \text{(b)}$$

Here $q(I) = p(I_r(I))$ is an effective probability of audit for taxpayers with income $I$.

Below we prove the theorem under condition

$$T(I) \le I - I_{\min} \text{ for any } I. \qquad (4.2)$$

If $q(I) > 0$ then (a) implies (4.2). We conjecture that this condition is not binding if $q(I) = 0$ and $I_r(I) \ne I$. In any case such constraint seems to be unrestrictive in our setting.

First consider the problem under (4.2) but without condition (2.1.4).

Let us study optimization problem with respect to audit rule under a fixed non-decreasing tax schedule $T$. According to Sanchez and Sobel (1993), the optimal audit rule is a solution of the problem

$$R(q(.)) = \int q(I)[(1+\delta_a)T_+^{'}(I)\mu(I) - c]dG(I) \to \max_{q(.)}$$

under constraints: $q(I)$ does not increase in $I$, $q(I) \in [0,(1+\delta_a)^{-1}]$,

$$R_g(q(.)) = \int q(I)(1+\delta_a)T_+^{'}(I)\mu(I)dG(I) \le \Delta EI. \qquad (4.3)$$

Proposition 2 in Sanchez and Sobel shows that there always exists a solution of the form:

$$q^*(I) = \begin{cases} 1/(1+\delta_a) & \text{if } I \in [I_L, I_1), \\ p & \text{if } I \in [I_1, \hat{I}), \\ 0 & \text{if } I \in [\hat{I}, I_H], \end{cases}$$

where $I_L \le I_1 \le \hat{I} \le I_H$, $\quad p \in (0, 1/(1+\delta_a))$.

This effective probability is generated by audit rule

$$p^*(I)=\begin{cases}\dfrac{1}{1+\delta_a} & \text{if } I\in[I_L,I_1),\\ \mathrm{p} & \text{if } I\in[I_1,I_2),\\ 0 & \text{if } I\geq I_2,\end{cases}$$

where $I_2$ meets equation $T(I_2)=T_{eff}(\hat{I})=T(I_1)+p(1+\delta_a)(T(\hat{I})-T(I_1))$.

This result easily follows from the linear programming theory: (4.3) may be considered as a linear programming problem with two constraints besides monotonicity of $q(I)$. Hence there exists solution $q^*(I)$ and corresponding $p^*(I)$ which has at most two jumps. Moreover, if $T_+^{'}(I)\mu(I)$ does not increase then there exists a solution with only one jump, that is a cut-off rule. Note that constraint (2.1.4) is not binding with respect to $T$ that meets (4.2) under any cut-off rule. Moreover, for any pair $(T(.),p(.))$ where $p(.)$ is a cut-off rule with a threshold $\bar{I}$ there exist an equivalent evasion-proof strategy $(\overline{T}(.),p(.))$ where $\overline{T}(I)=\begin{cases}T(I), & I\leq\bar{I}\\ T(\bar{I}), & I>\bar{I}\end{cases}$.

In order to complete the proof, let us show that for any permissible pair $(T,q)$ including 3-level effective audit probability in problem (2.1.2-2.1.3, 4.2) there exists permissible strategy $(T^*,q^*)$ where, for any $I$, $T^*(I)=I-I_{\min}$ (so $T$ is a concave tax schedule), $q^*$ is either 3-level or a cut-off audit rule and $R(T^*,q^*)\geq R(T,q)$.

Consider the problem with respect to $T$ under fixed $q$. We can rewrite it as follows:

$$R(T,q)=\int T_{eff}(I)dG(I)-c\int_{I_L}^{\hat{I}}q(I)dG(I)\rightarrow\max_{T(.)}$$

where $T_{eff}(I)=(1-p(1+\delta_a))T(I_1)+p(1+\delta_a)T(I)$ for $I\in(I_1,\hat{I}]$, (4.4)

$$T_{eff}(I)=T(I) \text{ for } I\leq I_1,\ T_{eff}(I)=T_{eff}(\hat{I}) \text{ for } I>\hat{I}$$

under constraints:

$$R_g(T,q)=\int T_{eff}(I)dG(I)\leq EI \tag{4.5}$$

and (4.2).

If we exclude constraint (4.5) then $T^*$ is a solution since $T_{eff}(I) \le T^*_{eff}(I)$ proceeding from (4.4, 4.2). If $(T^*, q)$ does not meet (4.5) then consider $R$ as a function of $\widehat{I}$ under fixed $T^*, I_1$ and $p$. For $\widehat{I}^{'} \le I_1$ let us define $q(I \,|\, \widehat{I}^{'})$ as a cut-off rule with threshold $\widehat{I}^{'}$. Note that $R$ is continious and increases in $\widehat{I}$. Thus there exists $\widehat{I}^{'} \in (I_1, \widehat{I})$ such that $R_g(T^*, \widehat{I}^{'}) = \Delta EI$ and $R(T^*, \widehat{I}^{'}) > R(T, \widehat{I})$ because

$$\int q(I \,|\, \widehat{I}^{'}) dG(I) = \int_{I_L}^{\widehat{I}^{'}} q(I) dG(I) < \int_{I_L}^{\widehat{I}} q(I) dG(I).$$

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