

LONG-RUN IMPLICATIONS OF POPULATION AGING FOR THE PENSION SYSTEM OF UKRAINE

Katerina Lisenkova

Economic Education and Research Consortium
klisiankova@eerc.kiev.ua

Igor Eremenko

Institute of Economic Research and Policy Consulting
eremenko@eerc.kiev.ua

Phone: 380 (44) 203-72-13

Address: 609/14-b Tsvetaeva st.
Kiev, Ukraine, 02232

Abstract

Given relatively short horizon of any government, demographic changes, that have long-run consequences, frequently get neglected in economic policies. Population aging has serious long run implications for economic development. This process will have very important effect on the efficiency of social security systems of many countries of the world.

Ukraine also faces problem of population aging. Today share of Ukrainian population that is eligible for pension benefits is about 30% and it will increase further. This has dangerous consequences for the existing pension system, which is Pay-As-You-Go type system. In this work we study the impact of the demographic development of Ukraine on the sustainability of its pension fund. For this purpose we adapt the economic-demographic model developed by Social Security Reform Project of the International Institute for Applied System Analysis (Austria). We perform simulations using 6 different scenarios.

We find that according to two out of three demographic scenarios Ukrainian Pension Fund becomes insolvent in 30 years if no changes are made to its arrangement. If pension eligibility age increased to 65 years for both sexes, than system would have positive balance for all demographic scenarios, but only provided that pension contributions will not be reduced. Sensitivity analysis confirms that under the reasonable assumptions about exogenous parameters Ukrainian pension system will not be able to sustain itself in the future due to the problem of population aging.

Introduction

Demographic changes have significant influence on the economic performance of the countries, regions and the whole world. One of the most widely observed demographic trends is population aging, i.e. increasing proportion of elderly generations in the age distribution of population. In developed countries population aging was the result of two demographic processes: declining fertility and increasing life expectancy. Post-soviet economies (including Ukraine) also face population aging, even though this countries experienced increasing mortality rates and declining life expectancy during the period of transition. Reduction in fertility is so significant that even declining life expectancy cannot compensate for it and proportion of old people in the population keeps rising.

Population aging has a number of serious consequences for the future of mankind. One of the most urgent ones is the impact of population aging on the social security system. The present paper is devoted to studying the consequences that aging population may have on the pension system of Ukraine.

Ukraine has experienced declining fertility rates through most of the 20th century. Its age and gender structure was also influenced by historic events (such as the Great October Revolution, Civil War, the famine of the early 1930's, World War II, the post-war "baby boom") that affected fertility and mortality patterns of Ukrainian population during this period. As a result, at the moment it has a very unusual population structure. But what common for Ukraine, other post-soviet countries, European countries, Japan, China, USA, and many other countries is that they all have fertility below replacement level (i.e., total fertility rate below 2.1) and face the problem of population aging, which has serious implications for their social security systems.

Currently Ukraine and most developed countries have a Pay-As-You-Go (PAYGO) pension system, under which pension benefits of current retirees are financed from the social security taxes paid by the current employees. This means that if old age dependency ratio (the number of retirees relative to number of workers in the economy) increases, then system will be unable to provide the same level of benefits to new retirees as before unless social security taxes are increased.

Now, Ukraine has relatively high social security taxes – 33% of total wage bill (32% paid by employer and 1% paid by employee). Thus, it has little room for tax increase. At the same time Ukrainian pension system, has low replacement ratio (ratio of average pension to average wage) of 38%. This pushes very large share of Ukrainian population below the poverty line (Zhalila, 2001).

Until now no major changes were made in social security system of Ukraine. But under the current arrangement it will be impossible to keep the system sustainable for a long time, taking into account limited ability of the government to rise payroll taxes and extremely low level of pensions.

The aim of this paper is to offer a quantitative assessment of the grave situation of the Pension Fund of Ukraine and to answer the question whether it has chance to survive in its current design if demographic trends persist. We conduct a number of social security simulations to project various scenarios of its further development. The model used for these simulations was developed by the International Institute for Applied System Analysis (IIASA, Laxenburg, Austria). This is an economic demographic model with a particular focus on the social security system. It allows us to determine the consequences for the Pension Fund under a variety of demographic and economic scenarios.

The set of scenarios for simulations was determined on the basis of two criteria: demographic dynamics (three types of population projections) and pension eligibility age (55-60 and 65-65 for women and men respectively). All together there are 6 scenarios.

The selected period of simulations is between 2000 and 2030. The model allows obtaining results on a number of parameters. But, for the purpose of this paper, particular attention is paid to the balance of public pension system. Obtained results track changes in Pension Fund balance over 30 years under 6 different scenarios. We also perform sensitivity analysis with respect to some exogenous parameters that might influence our results.

Review of relevant literature.

Perhaps, the most comprehensive analysis of an aging population and its economic implications is conducted by Schulz (1992). It is mainly concerned with influence that aging population has on American society and its social security system.

Another work that focuses on the American social security system in the context of aging population is presented by Anderson et al (2001). In this work authors adopt stochastic model of demographic change. This method of forecasting uses historical trends to model future economic and demographic trends. As a result they obtain less precise but more accurate conclusions about potential insolvency, because they are based on history. Results of this research present insolvency of the Trust Fund expressed as the probability that it will occur in a given year. Authors study the influence on the solvency of the Trust Fund of several policy measures: increase in the social security taxes, increase in the normal retirement age, and investing portion of the Trust Fund in the stock market. Studying the effect of each measure separately they find that very strong changes required to substantially extend the date of insolvency with a reasonable degree of certainty. A more effective alternative would incorporate some combination of measures. For example, they find that a modest increase in social security taxes and the normal retirement age accompanied with a moderate level of investment in the stock market achieves the goal of long-term solvency of the Trust Fund.

Social Security Reform Project of the International Institute for Applied Systems Analysis (IIASA, Laxenburg, Austria) has developed multiregional economic-demographic growth model that allows to model different scenarios of pension reform in a country that faces the problem of an aging population. MacKellar et al (1999) present a general description and algebraic structure of this model.

MacKellar et al (2000) present the structure and simulation properties of this model. For this purpose they take data for imaginary country or region and show how model performs under various assumptions. They also conduct uncertainty analysis changing various exogenous parameters and study their impact on the results of simulations. This way they study uncertainty in the coefficients of Cobb-Douglas production function, uncertainty in age-specific labor force participation rate, and uncertainty in age-specific average propensities to consume. They conclude that "reasonable

exogenous assumptions give rise to a reasonable long-run model solutions; when exogenous assumptions or model parameters are changed, the model performs sensibly on the baseline-vs.-alternative basis; and model projection results are reasonably robust ... to exogenous assumptions regarding household saving and labor supply; they are sensitive ... to the parameters of the core production function."

Dobronogov in a series of two papers used this model to study the Ukrainian pension system. Dobronogov (1998) uses simplified version of the model to study the impact of different scenarios of pension reform on the government debt. Scenarios are based on different share of informal sector in the economy, different duration of the transition from PAYGO to fully-funded pension system, and different pension eligibility age. The results show that, first, share of informal sector plays crucial role in the determining the outcome of the pension reform. Second, results show that the longer the duration of the transition the better chances for successful reform are. Third, according to simulations pension reform has no chances to succeed unless pension eligibility age will be increased.

In the another working paper Dobronogov et al (2000) focus their attention on the high degree of informalization of the Ukrainian economy. Authors include better described informal sector of the economy in the latest version of the model. Using this improved model Dobronogov et al try to determine the impact that social security system and its reform will have on the informal sector. Scenarios for simulations are based on the size of the fully-funded reformed pension system, on the method of the financing of the transition (debt-financed and tax-financed), and the degree of the public trust in the reform. The results of the simulation show that public trust plays very important role in the reform's impact on the informalization of the economy; debt-financed transition will facilitate decrease of the share of the informal sector in the economy; ceteris paribus, the larger the private fully-funded pension system, the smaller the informal sector of the economy. Thus, introduction of fully-funded component to the pension system may have positive impact on the reduction of the informal sector.

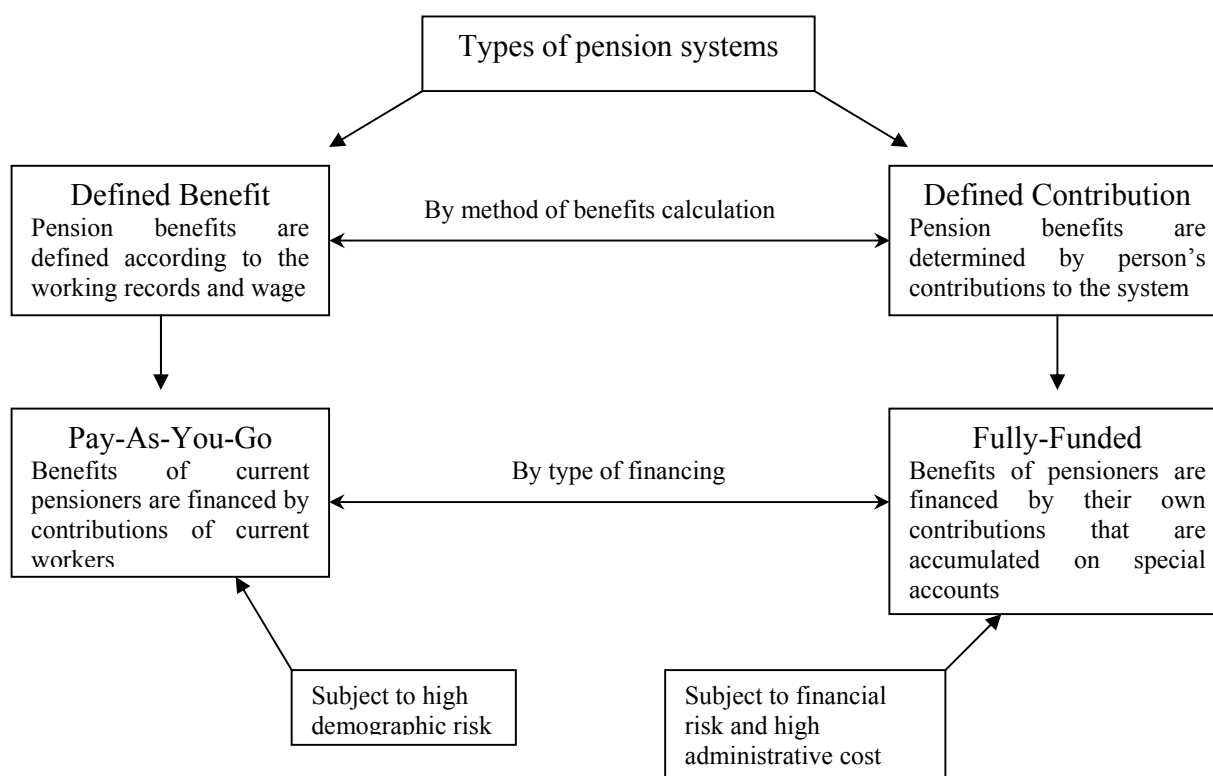
In may work we adapt the IIASA social security model. Unlike previous researches we concentrate our attention not on the impact that pension reform will have on the economy, but on the pension system itself and its balance, in particular. Distinguish feature of our research is its focus on the impact that demographic changes will have on the pension system and its possible reform.

Different Types of Pension System.

There are four ways to support the aged. First, they can continue to work and support themselves. Second, their family (in the wider sense of this word) can support them. Third, they can get pension benefits. Finally, they can live on their lifetime savings. Thus, pension is just one of the means of financing old age expenditures.

Pension benefits play an important role only in developed countries and in part of transition economies (those that previously formed the socialistic bloc). The rest of the world still follows traditional approach where children support their parents, who in turn try to work as much as they can (Dmitriev et al, 1998).

There exist several types of pensions systems. It is possible to classify them according to different principals. First, by the method of benefits calculation there are "defined benefit" (pension benefits of an individual are defined according to his wage and working record) and "defined contribution" (pension benefits are determined by person's contribution to the system) pension arrangements. Second, by type of financing one can distinguish "fully-funded" (FF, where pension contributions of current workers are accumulated on special accounts and after retirement, they receive benefits form this accounts in the form of annuity) and "pay-as-you-go" (PAYGO, where retirement benefits to current pensioners are financed by current workers) systems. In PAYGO pension systems benefits calculation is based on the defined benefit principle, while in FF pension systems benefits calculation is based on defined contribution principle (see Figure 1). Third, by the form of management there are publicly and privately managed systems. Currently in the world prevails publicly managed defined benefit PAYGO pension system (Dobronogov, Mayhew, 2000). Ukraine also has this type of pension system. Recently some countries introduced defined contribution FF pension systems (for example, Chile in 1981). While still others have combination of both FF and PAYGO components (Kazakhstan introduced it in 1998; Peru in 1993).

Figure 1. Types of pension systems

Defined benefit PAYGO pension system is the most vulnerable to the demographic shocks. In the context of population aging, it becomes unsustainable in the long-run. If benefits to current retirees are financed by current workers, then increase in the number of pensioners relative to the size of the labor force increases burden placed on the shoulders of the working generations. In addition to demographic risk, this type of arrangement gives incentives for workers to understate their earnings and pay lower social security taxes (free rider problem).

At the same time a defined contribution FF system is not subject to demographic risk. Every person that participates in it gets what he or she contributed to the system earlier plus the return (net of administrative costs) earned on those contributions over the period that money were held and invested by the pension fund. Demographic risk may be present only in the form of lower returns on contributions as the number of participants increase. This type of system also discourages evasion, and may stimulate higher national savings and development of the financial sector. FF system, however, is subject to financial risk, as contributions are invested in securities. The major concern with this system is that it may provide a rate of return which is less than market one. Another negative feature of the FF pension system is the high administrative and management costs. Finally, this system does not protect the most needy groups of population, those who had very low income level and were unable to save enough for retirement, and those who had long periods of unemployment during their working life. Under PAYGO system, they would usually receive some level of benefits.

Thus, each system has its pros and cons. Choosing between them (or any combination of both systems) we should take into account their particularities as well as the conditions in the country (one of which is the demographic situation).

Basic Conditions and Model Description.

Ukrainian pension system

- Ukrainian pension legislation

Legislative base for Ukrainian pension system was formed on the basis of the legislation of the former USSR. Now it consists of several laws. First and the most important one is the Law of Ukraine "On Pension Insurance" adopted in 1991. Other laws determine the level of pension payments for different categories of people. Funding of the pension system is regulated by the Law of Ukraine "On Tax for Mandatory State Pension Insurance" adopted in 1997

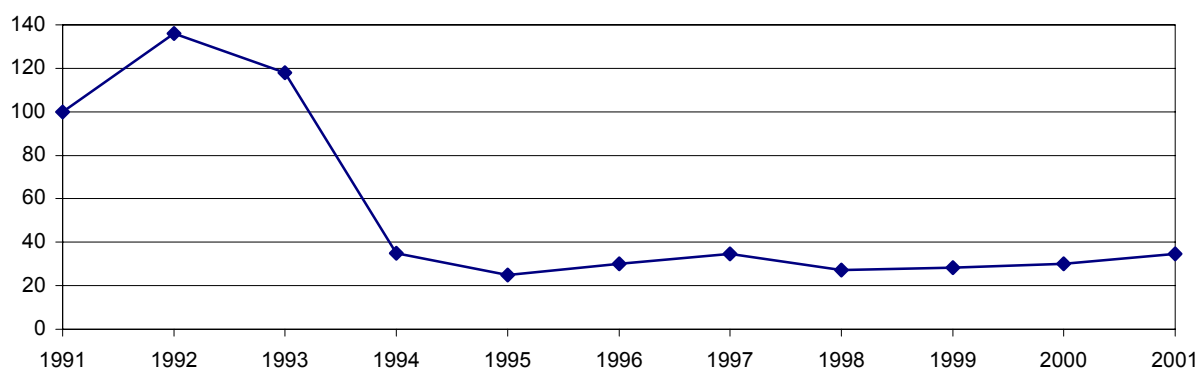
- Pension benefits level

For old-age pension eligible those workers who has reached pension age and have minimal required working record. Retirement age is 55 and 60 years for women and men respectively. Required working record is 20 years for women and 25 years for men. The level of an old-age pension is 55% of the average monthly wage but cannot be less than the minimal pension. For every additional year of work pension increases by 1% of the average monthly wage but this additional part cannot exceed 75% of pension. The minimum pension is determined by Verkhovna Rada (Parliament of Ukraine). The maximum level is three (in some cases four) times the minimum level. The level of labor pension is calculated on the base of the wage for the last two working years or any five years of work without interruption. The part of wage exceeding 10 minimal wages is not taken into consideration for pension calculations. That part of the wage that is less than 4 minimal wages is taken into full account. The fifth minimal wage enter the calculations with the coefficient .85, the sixth wage – with coefficient .7, the seventh – with coefficient .55, the eighth– with coefficient .4, ninth – with coefficient .25 and tenth – with coefficient .15.

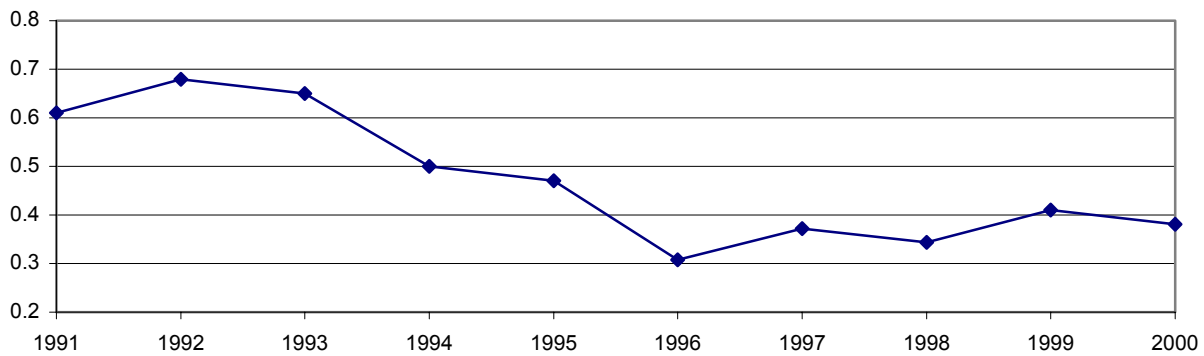
There also exist a number of categories of workers who are eligible for different types of privileges. For example, at the moment 28% of pensioners, who obtain old-age pensions had privilege to retire earlier than official pension eligibility age (Ivankevich, 2001). Another type of privileged categories of pensioners receives pensions that significantly exceed those of regular retirees. These categories of pensioners do not have upper bound for pension benefits as regular pensioners do (for example, judges, people deputies, etc.) (Nechaj, 2001).

During the transition period, pension benefits obtained by Ukrainian pensioners have deteriorated significantly. As shown in Figure 2, compared to the beginning of this process, pensions in real terms has declined more than two times. After 1995 we observe slight improvement, nevertheless current pension benefits are still more than two times lower than in 1991. But pensions have not only decreased in their absolute value, they have also decreased relative to wages. Thus, severe economic crises affected first of all and most of all the weakest. As we may see in Figure 3, replacement ratio has also declined during the period when pensions showed downward trend.

Figure 2. Index of average monthly pension (at the beginning of the period, 1991=100)



Source: State Statistical Committee of Ukraine, own calculations and Dobronogov (1998)

Figure 3. Replacement ratio (at the beginning of the period)

Source: State Statistical Committee of Ukraine, IMF Country Report, 2001, own calculations and Dobronogov (1998)

- Pension Fund Contributions

Pension Fund has two major sources of financing:

Employers transfer 32% of the total wage bill to the Fund;

Workers pay 1% of their wage.

The only type of investment that Fund can make if it has excess resources is purchase of the state bonds (Dobronogov et al, 2000).

- Necessity of the pension reform

Thus, pension system of Ukraine designed the way described above has number of problems.

First, employees pay very large pension taxes (32% of total wage bill), that gives them incentive to reduce officially reported wages and leads to tax evasion (Dobronogov et al, 2000). This fact imposes additional burden on pension system.

Second, pension eligibility age is very low. At the moment, life expectancy at retirement is approximately 22 year for women and 14 years for men (Ostanin, 2001). In addition, as mentioned above, there is relatively large number of people who are entitled for early retirement (representatives of about 20 professions) (Dobronogov et al, 2000).

Third, existing limits on maximum pension reduce differentiation of pension benefits. Person that never contributed to Pension Fund might receive approximately the same pension benefits as the one that contributed to it during the whole working life. This reduces incentives for workers to participate in official labor force and in Pension Fund formation. At the same time, according to current legislation, large part of pensioners has preferential treatment and has no limits on maximum pension (about 10% of new retirees). Such situation reduces the average level of benefits received by non-privileged categories of retirees and reduces their motivation to participate in this system even further. Preferential treatment of some categories of workers contradicts basic principals of pension reform, since pension benefits, that are paid out of Pension Fund should be determined exclusively by contributions that persons made to the system (Ivankevich, 2001).

Hence, to overcome problems of existing pension system and to improve needy position of the pensioners urgent pension reform is required.

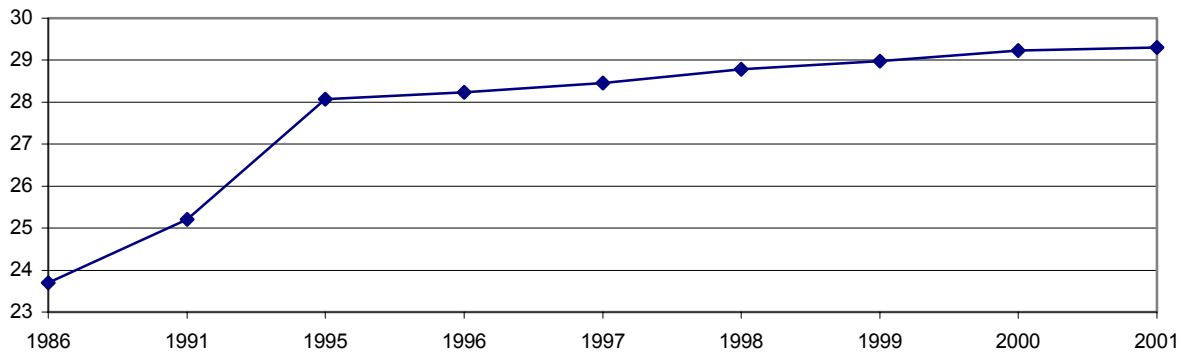
We should also mention, that demographic situation and its dynamics suggests that next decade will be relatively more favorable time for pension reform (see next section).

The problem of population aging in Ukraine

Difficult situation of the Pension Fund due to described reasons aggravated by growing number of pensioners (Figure 4). As a result, large share of the Ukrainian population lives below the poverty level. As of 01.08.2000 minimal old-age pension amounted to UAH 39, while officially reported subsistence level is UAH 311 (Zhalila, 2001). Another

illustration of the desperate position of pensioners is the fact that at the beginning of 2000 average pension level was UAH 68.91 while poverty level was UAH 90.7 (Ostanin, 2001).

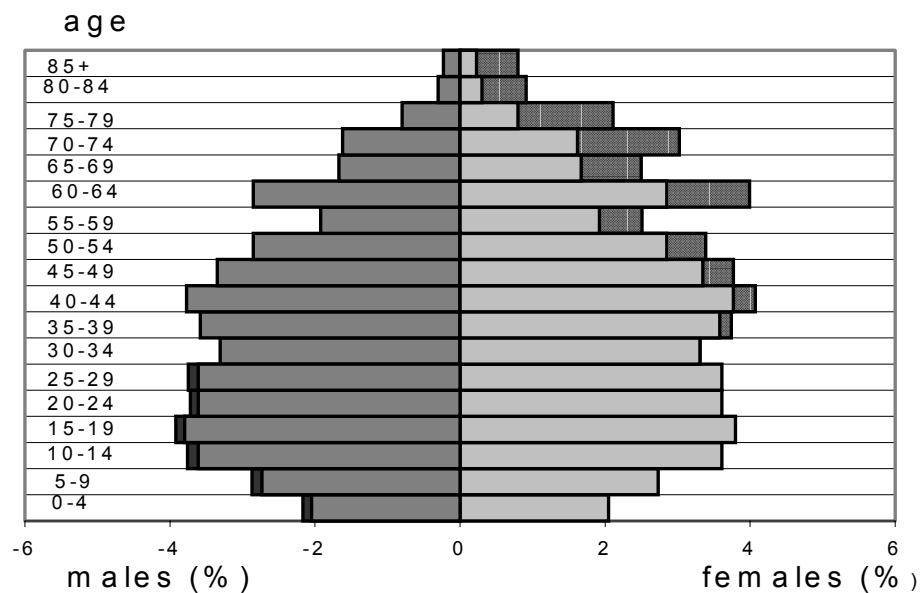
Figure 4. Ratio of the number of pensioners to total population of Ukraine (beginning of the period)



Source: State Statistical Committee of Ukraine, and own calculations.

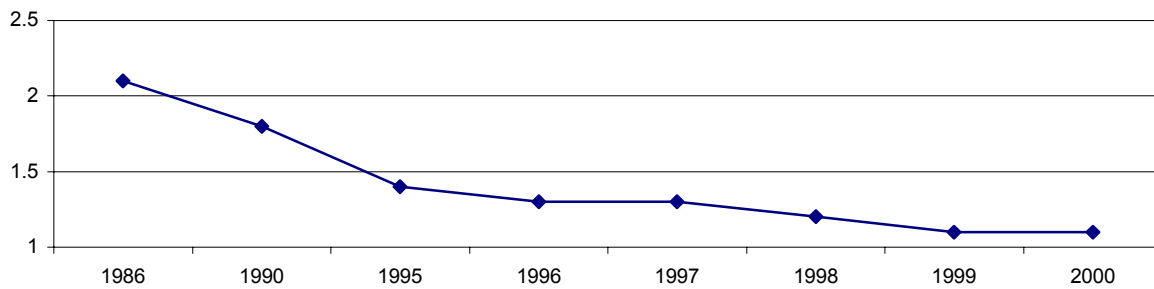
The fact that pension reform has to be designed in the context of population aging adds another challenge to this issue. Population age structure (Figure 5) shows clear tendency to population aging.

Figure 5. Population pyramid as of 01.01.2001.



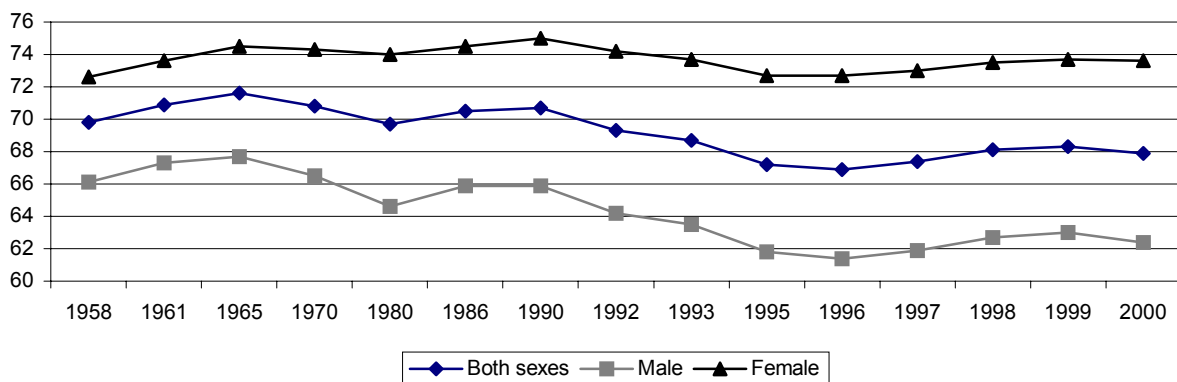
Source: State Statistical Committee of Ukraine, and own calculations.

The process of a population aging has two components. The first is declining fertility rate, which leads to smaller proportion of young generations in the population structure. The second is increasing life expectancy, which increases the number of elderly people. Most countries in the world experience sharply declining fertility rates. Ukraine, in particular, has total fertility rate below replacement level since 1986 (Figure 6).

Figure 6. Total fertility rate (beginning of the period).

Source: State Statistical Committee of Ukraine

Unlike developed countries, that experience very rapid growth in life expectancy due to advances in health care, transition economies had declining life expectancy during the transition period caused by economic and social instability. But at the moment, Ukraine also has tendency toward increase of life expectancy at birth after it experienced the lowest for the last 50 years level in 1995-1996 (66.9 for both sexes) (Figure 7). Hence, Ukraine is likely to have improvement on the life expectancy, which will fasten the process of population aging.

Figure 7. Life expectancy at birth (beginning of the period).

Source: State Statistical Committee of Ukraine

Taking into account the issue of population aging, we can predict that if current setting of the pension system is not changed, affordable replacement ratio will fall.

Analyzing structure and dynamics of the population we should pay particular attention to the so-called phenomenon of "demographic waves". Although, as we sad, demographic structure of Ukraine has tendency towards population aging, during the first decade of the third millennium it will experience slight improvement in the population structure. This fact can be explained by simultaneous action of several demographic trends. First of all, at the beginning of the century thin cohorts that were born during the WWII enter the retirement age. At the same, relatively large cohorts that were born during the first half of the 1980's reach working age and increase labor force (see Figure 5). Thus, till 2006 we can expect increase in the size and share of working age generations. Number of birth will also increase in the nearest future, as large cohorts of females that were born at the and of 1970's – beginning of 1980's will enter most active age for fertility (20-26 years old).

For this reason, the first decade of the 21st century is regarded "demographic chance" to implement pension reform in Ukraine.

Model description

In order to simulate future development of Ukrainian pension system, we adapt the economic demographic model based on the one developed by IIASA's (International Institute for Applied System Analysis) Social Security Reform Project (MacKellar and Ermolieva, 1999). This is a neoclassical two factor model with a particular focus on demographic development and pension system. The original model was modified to include large informal sector (which is the case with Ukrainian economy). This modification was presented in Dobronogov and Mayhew (2000) work where they studied the influence of different scenarios of pension reform in Ukraine on the informal sector of economy. In this work, we use one region version of the model. Complete algebraic structure of the model with its description is presented in Appendix 1.

The model is based on the UN System of National Accounts. Age-specific saving rates, labor force participation rates, and rates of distribution of savings between different types of investments are exogenous.

There are five types of agents in the economy: individuals by single-year age group, private unincorporated enterprises (PUE), firms (i.e. corporate enterprises), financial intermediaries (private pension system and other financial institutions), and government. The model tracks receipts and disbursements (i.e. net savings) of individuals, firms, and government.

There are two sectors of economy, the formal and the informal, each characterized by its own Cobb-Douglas production function.

$$GDP(t) = \alpha(1 + g)^t KTot(t)^\beta Emp(t)^{1-\beta} \quad (1)$$

where $KTot$ is total capital of the economy, Emp is total number of working population, α and β are exogenous coefficients of Cobb-Douglas production function, and g is the exogenous rate of total factor productivity growth.

Rates of return to factors are neoclassical:

$$R(t) = \beta \left[\frac{GDP(t)}{KTot(t)} \right], \text{ where } R(t) \text{ is time dependent interest rate, and} \quad (2)$$

$$\overline{WageRate}(t) = (1 - \beta) \left[\frac{GDP(t)}{Emp(t)} \right], \text{ where } \overline{WageRate}(t) \text{ is time dependent average wage.} \quad (3)$$

Informal sector in this model represents those agents that do not pay social security taxes from their earned income. At the same time it is possible that individuals pay social security taxes only from the part of their earnings. For the purpose of this model we call informal sector the share of total earnings from which social security taxes are not paid. It constitutes the share of labor that is employed in the informal sector (as model tracks only average wages), and the share of capital that attached to this labor and used for activity, from which social security taxes are not paid.

Capital in the formal sector ($KTotF$) consists of four types of capital. Residential capital ($KRsidential$), capital operated by PUE ($KPUE$), and capital operated by firms and held on behalf of households by private pension system ($KPvtPenSys$) and by other financial institutions ($KOFI$). Capital in the informal sector includes residential capital and capital of PUE. Firms operate only in the formal sector. Firms earn profits pay taxes and distribute dividends to holders of claims. Other financial institutions represent all types of financial intermediaries except for pension funds. In both sectors, rents earned on residential capital and profits of PUE accrue directly to the households.

Number of working people (Emp) depends on population, labor force participation rate and unemployment rate. Size of the population and its age-sex structure is taken from population projection.

Public pension system is pay-as-you-go system (PAYGO). Workers, who are employed in the formal sector, and their employees make pension contributions to it. After retirement, individuals receive public pension, calculated on the base of their working records and average wage earnings.

Contributions to the public pension system out of wages are

$$ContPubPenSysWageY(t,age) = PubPenSysContRate(t) WageY \quad (4)$$

The social security contribution rate $PubPenSysContRate(t)$ is assumed to be age-independent. No distinction is made between employees' and employers' contributions. Social security contributions out of entrepreneurial income are calculated similarly:

$$ContPubPenSysEntryY(t,age)=[PubPenSysContRate(t)EntryY((t,age)) \quad (5)$$

Let $BenEntPubPenSys(t, age, RtrmntDuration)$ be the social security benefit entitlement for the average person aged age , who retired $RtrmntDuration$ years ago, where we assume that $BenEntPubPenSys(t,age,0) = 0$. The pension for persons entering retirement is computed according to the formula:

$$BenEntPubPenSys(t, age, 1) = \frac{ReplRate(t) \left(\sum_{j=1}^{age-14} LabForcePartRate(t-j, age-j) \right) \sum_{k=1}^{Re fPeriod} WageRate(t-k, age-k)}{age-14 \quad Re fPeriod} \quad (6)$$

$ReplRate(t)$ is a coefficient of proportionality which translates earnings into an initial pension entitlement. This replacement ratio is applied to the average labor force participation rate between age 15 and age (assuming $age \geq EligAge$ (eligibility age)) times average annual earnings during the $RefPeriod$ (reference period) years prior to retirement.

System balance at time t is calculated as follows.

$$BalPubPenSys(t) = BalPubPenSys(t-1) + (1+r) BalPubPenSys(t-1) + ContPubPenSys(t) - BenPubPenSys(t) \quad (7)$$

There is defined contribution private pension system ($PvtPenSysDC$), where pension benefits are calculated on the basis of workers contribution to the system and age to which the person is expected to live.

There are six sources of household income. They are wages, profits earned on capital operated by PUE's, imputed rents from residential capital (housing services), dividends on capital operated by firms, public pension benefits, and private pension benefits.

In addition to disposable income, resources available for household consumption include capital transfers, which take the form of sales of assets accumulated during the work life in old age and inheritance of assets by the surviving population.

Thus, the model tracks accumulation of assets during the working age and their depletion after retirement.

One-region version that includes informal sector requires data on 93 parameters to perform simulations and gives results on 311 parameters (complete list of parameters is available from the authors).

Set of scenarios

In this work we analyze the effect that population aging has on solvency of public pension system. In fact, Ukrainian Pension Fund cannot become bankrupt in a literal sense. Indeed, in 1995 and 1996 benefits paid out of Pension Fund exceeded contributions to the Fund. But in this situation state budget covered the difference (in IIASA model deficit of the public pension system is one of the composite parts of the government debt). Thus, speaking about Pension Fund solvency, we mean non-negative balance on the public pension system. In any case pension system by design is assumed to sustain itself and state budget cannot cover system debts forever.

Scenarios used for simulation are defined according to two criteria (see Table 1).

The first is the type of population projection that is used for simulations. There are three of them:

- First population projection (high fertility and low life expectancy assumptions are used);
- Second population projection (medium fertility and medium life expectancy assumptions are used);
- Third population projection (low fertility and high life expectancy assumptions are used);

The second criterion is pension eligibility age:

- Remains unchanged (55 for women and 60 for men);

- Will be increased to 65 for both genders till 2010.

Hence, all together we have 6 scenarios.

Table 1. Set of scenarios

Number of scenario	Type of population projection	Pension eligibility age
1	First population projection (high total fertility rate, low life expectancy)	55 for females, 60 for males
2		65 for both genders
3	Second population projection (medium total fertility rate, medium life expectancy)	55 for females, 60 for males
4		65 for both genders
5	Third population projection (low total fertility rate, high life expectancy)	55 for females, 60 for males
6		65 for both genders

Population projections and simulation results

Population projections

All population projections are produced using fertility and life expectancy assumptions made by IASA population project for Former Soviet Union countries (including Ukraine) (Lutz et al, 1996).

In this work, we take only assumptions about fertility and mortality levels assuming migration out. This is done because of the high degree of uncertainty about migration level and direction, as it depends not only on the economic and political conditions in the country in question, but also on the economic and political conditions in all other countries. While we neglect migration in our work this can be a big issue for the development of pension system, which requires further investigation. Assumptions about the total fertility rate (TFR) and the life expectancy at birth are presented in Table 2.

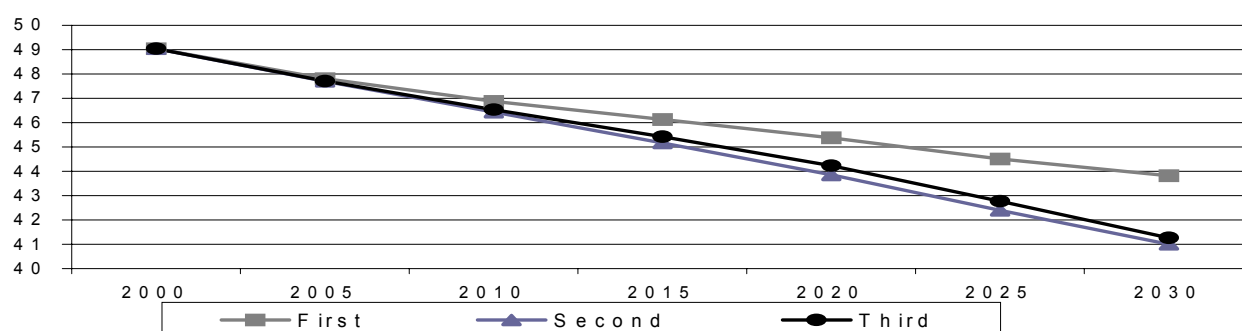
Table 2. Assumptions about total fertility rate and life expectancy at birth.

European FSU region		2000	2030-2035			2045-2050		
		Actual	Low	Medium	High	Low	Medium	High
Total fertility rate		1.1	1.3	1.7	2.1	1.4	1.7	2.1
Life expectancy at birth	Male	62.4	62.4	68.6	76.1	63	70	77
	Female	73.6	76.6	80.3	84.1	77	81	85

Second column of Table 1 presents actual level of the TFR and life expectancy at birth reported by the State Statistical Committee. For 2030-2035 and 2045-2050 projected values are presented. We assume that between 2000 and 2030, and 2030 and 2045 the TFR and the life expectancy at birth smoothly converge to the corresponding projected level.

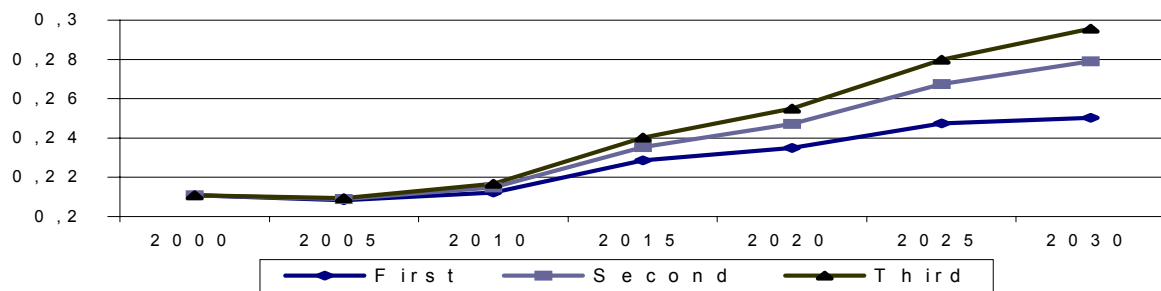
To make population projection We used computer program Demproj. On the basis of initial age-sex distribution and assumptions about the TFR, age distribution of fertility, and life expectancy at birth it allows to project size and composition of the population for 50 years. Results of projections are presented in Figure 8.

Figure 8. Total projected population of Ukraine according to different demographic scenarios (mln).



As can be seen in the figure projected population of Ukraine steadily falls according to all demographic scenarios. For the First population projection decline is not that rapid and projected population in 2030 is almost 3 millions larger than for other types of projections. This can be explained by higher TFR and favorable population structure. As was mentioned above, during the next 10-15 years Ukraine will have large cohort of fertile females that were born during 1980's, years that were characterized by relatively high fertility levels. When those cohorts will have children, relative number of newly born children will increase. We also should take into account that age-specific mortality rates are lower in younger generations. Thus, we can expect relatively larger total population according to the First population projection, which is based on high TFR and low life expectancy assumptions, than, say, that of the Third population projection, which is based on low TFR and high life expectancy.

Figure 9. Proportion of the total population eligible to pension benefits according to different demographic scenarios (pension eligibility age 55/60).



Common characteristic of all types of population projections is increasing proportion of the population eligible for the pension benefits. It is described in Figure 9. The Third population projection has the largest proportion of population eligible for pension benefits due to two reasons. The first is that it is based on the assumption of high life expectancy, which means larger number of people eligible to pension benefits. The second is that according to the Third population projection Ukraine will experience the lowest TFR, which means smaller size of young generations. Both this factors results in the larger proportion of elderly generations.

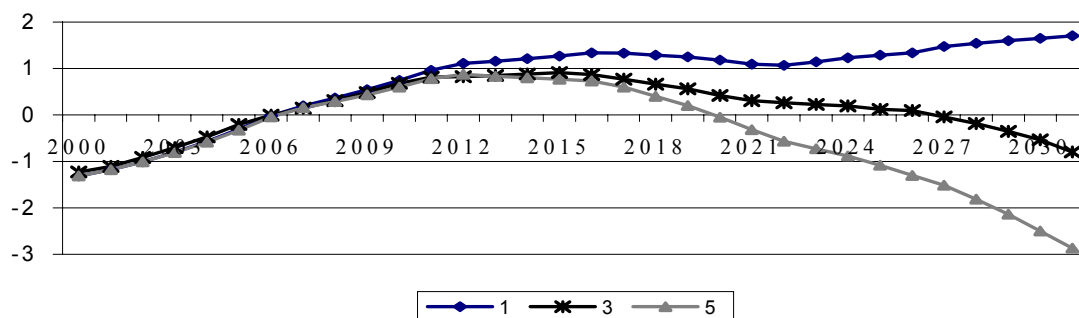
It is also possible to notice that during the first 5-6 years of projections proportion of population eligible to pension benefits according to current legislation will slightly decrease. This fact can be explained by the "demographic waves" described in the previous chapter.

Results of the simulations

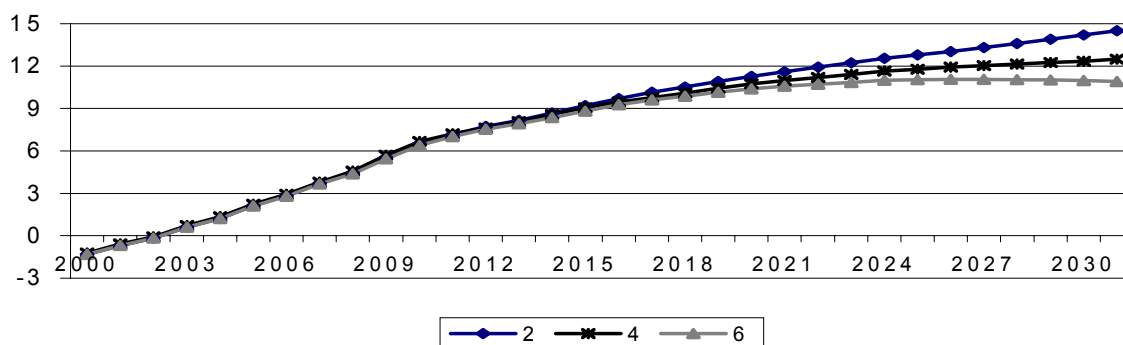
The simulations are made for the period from 2000 till 2030. Model allows to obtain results on the number of parameters, but here particular focus is on the balance of public pension system. Results of the simulation are presented in Figure 10.

Figure 10. Balance of the Public Pension system from 2000 till 2030 for different scenarios (bln of UAH).

a) Results for scenarios 1, 3, and 5



b) Results for scenarios 2, 4, and 6



According to two out of six scenarios Pension Fund will go bankrupt during the next 30 years. Both these scenarios are based on the present pension eligibility age. Results for the scenario number one show positive balance of the public pension system, although it is also based on the current pension eligibility age. But this scenario is based on the First population projection, which has high TFR assumption and low life expectancy, which means that Ukraine will experience slower growth on the proportion of population eligible for pension benefits. But, as was mentioned above, Ukraine will likely have tendency toward increasing but not decreasing life expectancy. Thus, more attention should be paid to the third scenario, which is based on medium assumption about fertility and mortality levels and has better chances to be closer to the reality.

Some limitations of the model also make situation with Pension Fund look better than it actually is. As shown in Figure 9 proportion of people eligible for pension benefits in 2000 is about 21% of total population. At the same time Figure 4 suggests that in 2000 Ukraine had about 29% pensioner in the structure of total population. This is the result of the fact that some pensioners start to receive pension benefits before the official pension eligibility age for various reasons (disability, privileges, etc.). Although model tries to account for those pensioners that obtain disability or survivors pension, it does not consider possibility of earlier retirement, which, as was mentioned, has very large scale in Ukraine. Thus, actual number of pensioners *ceteris paribus* would exceed that calculated by the model. This fact unambiguously acts in favor of the public pension system balance.

Another fact of reality that is not taken into account in our research is migration. As was mentioned above, we neglected this issue because of high degree of uncertainty attached to it. But still, we have to keep in mind that at the moment Ukraine has net emigration (Lutz et al, 1996 also assumes that Former Soviet Union region will have net emigration in the future). If this tendency will persist in the future the size of Ukrainian labor force is likely to decrease due to emigration in larger proportion than the number of pensioners, as emigrants are mostly young, high-skilled and highly motivated people. Hence, our population projections are likely to be more favorable for Ukrainian pension system, than those that would take migration into account.

Taking all these facts into account, we can conclude that, according to these simulations, there are very big chances that Pension Fund of Ukraine will go bankrupt during the next 30 years if no changes are made in the arrangement of pension system.

Panel b) on Figure 10 presents results of simulations for scenarios 2, 4, and 6, which assume increase in pension eligibility age to 65 years for both sexes. In this case for all population projections system has large positive balance. Results look very promising. However, if we assume that pension eligibility age will be increased by 5 years for males and by 10 years for females, we need to realize that contribution rate, most likely, will be reduced. Decrease in contribution rate would reduce total contributions to the system and, consequently, would decrease balance of the system. We investigate this possibility in the next section.

Sensitivity analysis

Results that are presented in the previous section can be sensitive to the values of exogenous parameters. For sensitivity analysis we expand the period of simulations to 50 years because in some cases for changes to become obvious it takes longer period.

First, let's see what influence on the balance of the public pension system would have simultaneous increase in pension eligibility age and reduction in contribution rate. For this purpose we take scenario 4 (2nd population projection and increased pension eligibility age) and change contribution rate from 32% to 22%. Current contribution rate is 32 % of total wage bill. Results of simulation are presented in Figure 11. As we see, if contribution to the pension fund would be reduced pension fund would have problems even if pension eligibility age is increased.

Second, let's see how balance of public pension system is effected by the share of employed labor that does not pay pension taxes (we should remember, that as model tracks only average wages this parameter presents share of earned income from which pension taxes are not paid). For this purpose we take scenario 3 (2nd population projection, current pension eligibility age) and change share of labor that doesn't make contributions to the pension system from 25% to 45% (baseline scenario is 40%). Results are presented in Figure 12. Results are very sensitive to this exogenous parameter, but even with the lowest level of 25% public pension system becomes insolvent within next 40 years.

Figure 11. Balance of the public pension system under assumptions of scenario 4 with different levels of contributions to the pension fund (bln of UAH) (baseline scenario is 32%)

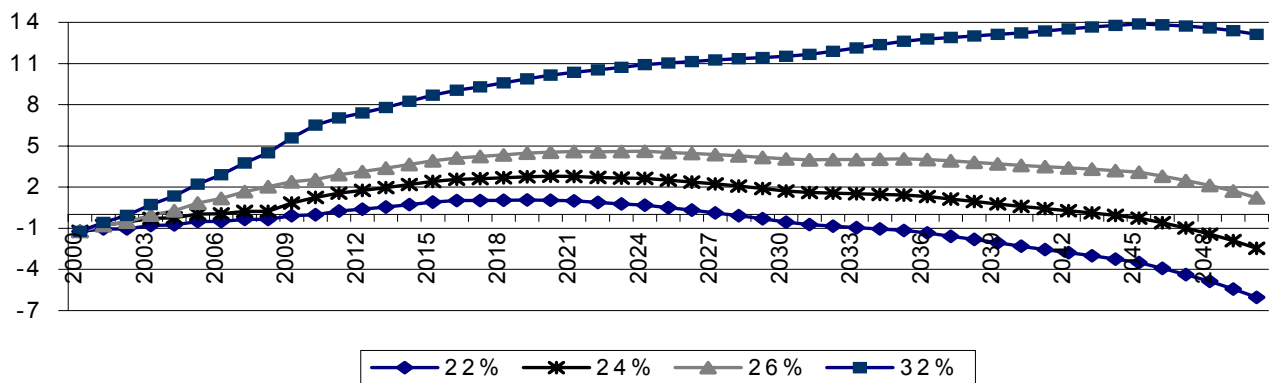
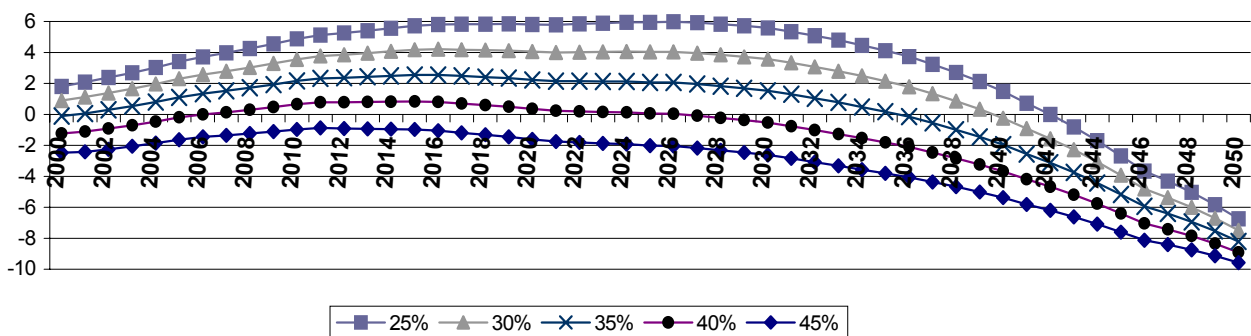
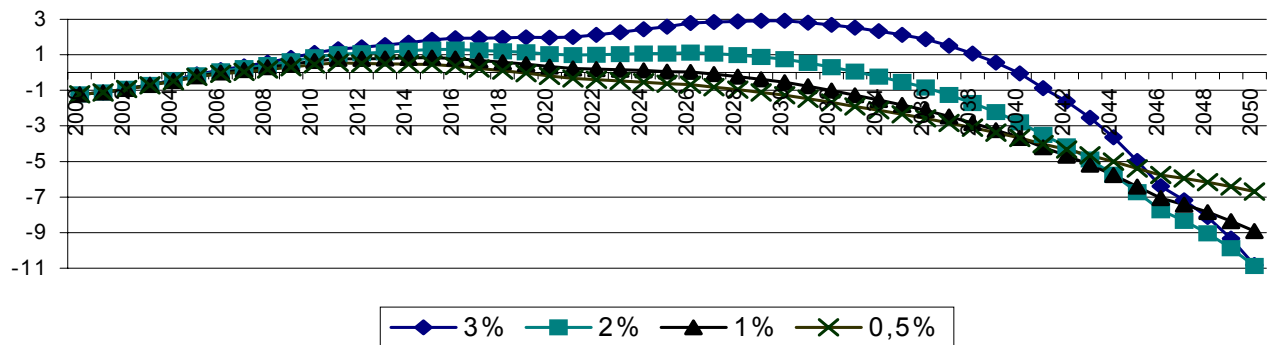


Figure 12. Balance of the public pension system under assumptions of scenario 3 with different shares of employed labor that does not make contributions to the pension fund (bln of UAH) (baseline scenario is 40%)



Third, let's see what effect total factor productivity growth has on the balance of the public pension system. For this purpose we take scenario 3 and change assumption about total factor productivity growth from 0.5% to 3% per year (baseline scenario is 1%). Results of the simulations are presented in Figure 13. As the figure reveals, even under the assumption that average total factor productivity growth during the next 50 years will be 3% resources of the public pension system will be depleted within next 40 years.

Figure 13. Balance of the public pension system under assumptions of scenario 3 with different levels of the total factor productivity growth (bln of UAH) (baseline scenario is 1%).



Hence, it is possible to see that most of the extensions to the original scenarios also suggest that during the next 50 years Ukrainian pension system in its current setting, most likely, will not be able to sustain itself.

Conclusion

Ukraine has Pay-As-You-Go public pension system. Under this type of arrangement benefits to current retirees are paid out of contributions of current workers. This type of system is subject to the high demographic risk. If number of workers decline and number of pensioners increase this type of system is unable to sustain itself in the long-run.

Ukraine, as well as many other countries, currently faces problem of population aging. In case of Ukraine, this is primarily the result of the dramatically low level of fertility, which it experienced during the last half of the 20th century. This will have serious negative impact on the Ukrainian pension system if it keeps its current setting.

During the next 5-10 year demographic situation in Ukraine will improve due to the effect of the “demographic wave”. But after that period it will worsen very rapidly. That is why specialists call next decade the “demographic chance for pension reform”. Results of this work show how it is crucial for Ukrainian pension system to use this “demographic chance” and start serious pension reform in the nearest future.

Simulations that are performed by means of demographic-economic model developed by the International Institute for Applied System Analysis (IIASA, Laxenburg, Austria) reveal that if no changes are made to the system than it becomes insolvent during the next 30 years according to the two out of three demographic scenarios.

Results are rather sensitive to the assumptions about the share of employed labor that does not contribute to the pension system and about the level of total factor productivity growth. But even under the most favorable out of reasonable range of assumptions about these two parameters simulations show that public pension system will become insolvent in about 40 years.

If pension eligibility age is increased, system has growing positive balance, but only under the assumption that contribution rate will not be reduced. But system goes bankrupt within 50 years even with moderate reduction from 32% to 26% of wage bill.

Thus, these results reveal that if current demographic trends persist prospects of Ukrainian pension system are rather bleak. Increase of pension eligibility age can serve as partial solution but only in the short-term.

The nature of the problem of population aging and its impact on the pension system suggests two major directions, in which we can seek for solution.

One possibility is to reduce the effect of demographic trends on the pension system. Population aging, that has significant effect on Pay-As-You-Go type of systems, has no effect on the fully-funded systems. Hence, in principle switch to fully-funded system could solve this problem. However, this is not that simple, as transition from one type of

system to another is very costly. For transition period both systems should be financed. Workers have to accumulate funds for their retirement in fully-funded system. At the same time, pension benefits to retirees, who contributed to Pay-As-You-Go system during their working career, have to be paid out of old system. There are two possible types of transition. One is tax financed transition, when both systems are financed from internal sources, i.e. by taxes. Another is debt financed transition, when government finds external sources to finance transition, for example, borrows from international financial organizations (as was the case with Kazakhstan). Both types of transition may have very significant negative consequences for economic situation in the country. We also should remember that although fully-funded system is not subject to demographic risk it has other negative features. In any case, transition to fully-funded system requires additional research, and, perhaps, some combination of two systems would do a better job than any of them by itself.

Another possibility is to find the way to change demographic structure and dynamics. There are two ways to do it. First of them is implementation of fertility affecting policy. With this suggestion, however; one should keep in mind that, first of all, it is very difficult to affect fertility decision, second, it is very costly, third, it is not likely to change fertility behavior very much, and finally, this measure can have only long-term effect on labor force and pension system.

Second way could have desired short-term effect, but also has other arguable features. It is immigration. Having active immigration policy, Ukraine could at least partially solve population aging problem in the short-term, as immigrants are mostly young and active people. Immigration could also have other positive effects on economic and demographic situation in the country (development of international trade, effect on the genetic pool). The effect that migration could have on the Ukrainian pension system requires independent research. In any case immigration stays political issue rather than economic one.

Thus, this research reveals that if demographic trends will not change dramatically in the nearest future, Ukrainian Pay-As-You-Go public pension system will not be able to serve its function in the full magnitude due to the problem of population aging. Analysis shows that increase in the pension eligibility age would partially resolve the problem. Other major options, such as transition to fully-funded system, stimulation of fertility and implementation of active migration policy are suggested for additional research.

References

- Anderson M., Tuljapurkar S., Lee R. D., 2001, *Chances are...Stochastic Forecasts of the Social Security Trust Fund and Attempts to Save It*, Working Paper, Michigan Retirement Research Center.
- Dmitriev M., Travin D., 1998, *Pension reform in Russia: reasons, content, perspectives*, St. Petersburg, Norma.
- Dobronogov A., Mayhew L., 2000, *Pension Reform in a Highly Informalized Post-Soviet Economy*, Working Paper, Laxenburg, Austria, International Institute for Applied System Analysis, Available from www.iiasa.ac.at/.
- Dobronogov A., 1998, *System Analysis of Social Security in a Transition Economy: The Ukrainian Case*, Working Paper, Laxenburg, Austria, International Institute for Applied System Analysis, Available from www.iiasa.ac.at/.
- IMF Country Report, 2000, Available from <http://www.imf.org/>.
- Ivankevich V., 2001, *Pensijna Reforma v Ukraini (The Pension Reform In Ukraine)*, Aspecty Pratsy (Aspects of Labor), 2.
- Kurilo I., 2001, *Pro Mediko-Demografichni Chynnyly Formuvannia i Vidtvorennia Trudovogo Potencialu Nasellenia Ukrainy (About Medico-Demographic Factors of Formation and Reproduction of Labor Potential of the population of Ukraine)*, Aspecty Pratsy (Aspects of Labor), 8.
- Kuznetsov A., Ordin O., 2001, *Pension Reform in Russia a General Equilibrium Approach*, Working Paper, Economic Education and Research Consortium.
- Lee R., 2000, *Sustainable Social Security: What it Would Cost?* Available from <http://www.ceda.berkeley.edu/papers/>.
- Lutz W., Sanderson W.C., Scherbov S., 1996, *Probabilistic World Population Projections Based on Expert Opinion*, Working Paper, Laxenburg, Austria, International Institute for Applied System Analysis, Available from www.iiasa.ac.at/.
- MacKellar L., Ermolieva T., Westlund A., 2000, *A Social Security Forecasting and Simulation Model*, Working Paper, Laxenburg, Austria, International Institute for Applied System Analysis, Available from www.iiasa.ac.at/.

- MacKellar L., Ermolieva T., 1999, The IIASA Social Security Reform Project Multiregional Economic-Demographic Growth Model: Policy Background and Algebraic Structure, Working Paper, Laxenburg, Austria, International Institute for Applied System Analysis, Available from www.iiasa.ac.at/.
- Nechaj A., 2001, Ogljad Typiv Pensijnogo Zabespechennia za Zakonoproectom "Pro Nederzhavne Pensijne Zabespechennia" (Review of types of the pension funds according to the law-project "On the Private Pension Funds"), *Aspeky Pratsy (Aspects of Labor)*, 5.
- Ostanin D., 2001, Social Security System Reform: Risk Sharing Properties. The Case of Ukraine, Unpublished MA Thesis. Kiev, Kyiv-Mohyla Academy, EERC
- Schulz J.H., 1992, *The Economics of Aging*, Westport, Connecticut, Auburn House
- State Statistics Committee of Ukraine, 1997, 2000, 2001, *Statistical Year Book of Ukraine*, Kyiv, Ukraijnska entsiklopedia
- Zhalila Y., 2001, Vid Social'noho Zabespechennia do Social'noyi polityky (From Social Maintenance to Social Polisy), Kyiv, Satsanga.

Appendix 1.

The IIASA model: algebraic structure (MACKELLAR ET AL 2000)

In the following pages, the algebraic structure of the IIASA model is presented and its economic logic is described. Here the single-region version of the model is described. MacKellar and Ermolieva (1999) present an earlier version of the model in two-region form.

1. Population, labor force, and employment

1.1 Population

Population is divided into age groups $age = 0, MaxAge$ where $MaxAge$ is the beginning year of the terminal age category (for example, $100 = MaxAge$ if the terminal age category is $100+$). Single deterministic demographic scenario consisting of population by single-year age groups is loaded from another source.

Total population is the sum over age groups

$$Pop(t) = \sum_{age=0}^{MaxAge} Pop(male, t, age) + \sum_{age=0}^{MaxAge} Pop(female, t, age) \quad (1.1.1)$$

1.2 Labor force and employment

Total labor force is the sum over age groups:

$$LabForce(t) = \sum_{age=15}^{MaxAge} LabForce(t, age) \quad (1.2.1)$$

where

$$LabForce(t) = Pop(t, age) LabForcePartRate(t, age) \quad (1.2.2)$$

Age-specific labor force participation rates are exogenous assumptions, as are unemployment rates:

$$Emp(t) = \sum_{age=15}^{MaxAge} Emp(t, age) \quad (1.2.3)$$

$$Emp(t) = LabForce(t, age) [1 - UnempRate(t, age)]$$

2. Capital and the nature of claims

There are four types of capital: residential capital ($KRes$), capital operated by private unincorporated enterprises ($KPUE$), capital operated by firms and held on households' behalf by the private pension system ($KPvtPenSys$), and capital operated by firms and held on households' behalf by other financial institutions ($KOFI$). Also implicitly assigned to OFI are households themselves to the extent that they individually hold claims on capital operated by firms (i.e., claims that are disintermediated). Firms operate capital, either distributing or reinvesting earnings; they do not own shares in other firms. Financial claims on the capital operated by firms are held on behalf of households by institutions ($PvtPenSys$ and OFI) which collect and distribute dividends. No distinction is made between equity and debt claims.

The private pension system is divided into two components, one of which is a partially funded defined benefit (DB) system (*PvtPenSysDB*) and the other of which is a fully funded defined contribution (DC) system (*PvtPenSysDC*). Voluntary retirement savings accumulated outside pension funds are implicitly assigned to *PvtPenSysDC*. Corresponding to each of the four types of capital is an age-specific capital accumulation equation, which tracks the accumulation of assets for each cohort as it ages. There is a structural difference between the dynamics of *KPvtPenSys* and the dynamics of the other three asset classes. Funds flow into *PvtPenSys* only through payroll deductions (including deductions from entrepreneurial income) on behalf of system participants. Dividends earned on assets held by the *PvtPenSys* remain within the system. By contrast, savings of all origins, not just captive retirement-related savings, are invested in *KOFI*, *KRes*, and *KPUE*. Dividends earned on assets held by *OFI* accrue to households, instead of being retained by the intermediary, and may be allocated to consumption at any point during the life cycle, as may profits accruing to *KPUE*. Implicit rents on *KRes* are consumed, by assumption, in their entirety; equivalently, all housing is assumed to be owner-occupied. If saved, dividends earned on assets held by *OFI* may remain within *OFI*, or be allocated to residential investment or investment in capital operated by *PUE*.

All capital ultimately belongs to households. As described in Section 8, each single-year age-cohort is tracked as it accumulates capital during its working life and draws it down during retirement. Total assets of a cohort in a given year are

$$K_{tot}(t, age) = KPvtPenSys(t, age) + KRes(t, age) + KPUE(t, age) + KOFI(t, age) \quad (2.1.)$$

which expresses a cohort's wealth as the sum of pension- and non-pension wealth.

3. Output and rates of return to factors

3.1 GDP, wage rate, and rate of return to capital

Gross domestic product (*GDP*) is given by a Cobb-Douglas production function

$$GDP(t) = \alpha(1 + g)^t K_{Tot}(t)^\beta Emp(t)^{1-\beta} \quad (3.1.1.)$$

where g , the rate of total factor productivity growth, is exogenous. Rates of return to factors are neoclassical:

$$R(t) = \beta \left[\frac{GDP(t)}{K_{Tot}(t)} \right] \quad (3.1.2.)$$

$$\overline{WageRate}(t) = (1 - \beta) \left[\frac{GDP(t)}{Emp(t)} \right] \quad (3.1.3.)$$

where R is the gross profit rate, including depreciation and indirect taxes net of subsidies; and $\overline{WageRate}$ is average (over age groups) employee compensation, including social insurance contributions (contributions to public and private pension schemes).

In order to net depreciation and indirect taxes out of the rate of return to capital, we define

$$r(t) = R(t) - \frac{IndTaxRate(t)GDP(t)}{K_{Tot}(t)} - DeprRate(t) \quad (3.1.4.)$$

where $IndTaxRate$ is defined with respect to GDP and $DeprRate$ is the depreciation rate. The advantage of netting out depreciation and indirect taxes immediately is that we can henceforth ignore them in calculating income, outlay, and net savings.

3.2 Age-specific wage rates

In a model with age-structure detail, we require a procedure to ensure that the average wage rate calculated from the marginal productivity condition above equals the average wage rate calculated by summing across age groups. In practice, this means that age-specific wage rates must be adjusted to be consistent with the overall average wage rate. We have approached this problem by defining a scale factor $\sigma(t, age)$ and then calculating age-specific wage rates as

$$WageRate(t, age) = \sigma(t, age) \overline{WageRate}(t) \quad (3.2.1.)$$

$\sigma(t, age)$, a proxy for human capital, is specified to be logarithmic in age and parameterized so that wages rise rapidly in the twenties and thirties, the average wage over the life cycle is earned at approximately age 45, and there is little increase after 55. The required consistency condition is

$$\overline{WageRate}(t) = \frac{\sum_{age=15}^{MaxAge} Mult(t, age) \sigma(t, age) \overline{WageRate}(t) Emp(t, age)}{\sum_{age=15}^{MaxAge} Emp(t, age)} \quad (3.2.2.)$$

To simplify the problem, let $Mult(t, age)$ to be the age-invariant

$$Mult(t) = \frac{\overline{WageRate}(t) \sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age) \overline{WageRate}(t) Emp(t, age)} = \frac{\sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age) Emp(t, age)} \quad (3.2.3.)$$

This variable can be interpreted as total "nominal" employment relative to total "effective," i.e. human capital adjusted employment. Then the identity required for consistency may be rewritten

$$\overline{WageRate}(t) = \frac{\left. \sum_{age=15}^{MaxAge} \left\{ \frac{\sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age) Emp(t, age)} \sigma(t, age) \overline{WageRate}(t) Emp(t, age) \right\} \right\}}{\sum_{age=15}^{MaxAge} Emp(t, age)} \quad (3.2.4.)$$

Moving age-invariant terms outside the braces,

$$\overline{WageRate}(t) = \frac{\frac{\sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age) Emp(t, age)} \overline{WageRate}(t) \sum_{age=15}^{MaxAge} \sigma(t, age) Emp(t, age)}{\sum_{age=15}^{MaxAge} Emp(t, age)} \quad (3.2.5.)$$

which will clearly always hold true. Therefore, we calculate

$$\overline{WageRate}(t, age) = Mult(t) \sigma(t, age) \overline{WageRate}(t) \quad (3.2.6.)$$

where $Mult(t)$ is as defined above.

4. Income, capital transfers, outlay, and net saving of households

The articulation of income flows elaborated below has two main purposes. The first is to disaggregate income and consumption by age. The second is to make explicit the role of the private pension system in saving and the allocation of capital.

4.1 Income

The sources of household income are wages, imputed rents from residential capital, profits which accrue to capital operated by unincorporated enterprises, dividends distributed from earnings on capital operated by firms, public social security system benefits, and private pension benefits.

4.1.1 A note on taxation

The taxation of factor incomes in this model follows three simplifying assumptions. First, factor income is taxed once and only once, when it is earned. Thus, dividend income is not taxed because profits have already been taxed at the level of the firm; similarly, there is no capital gains tax when assets are sold because capital gains reflect profits which have already been taxed. Second, no distinction is made from a taxation point of view between different types of capital: profits on capital operated by

firms, capital operated by private unincorporated enterprises, and the imputed services of residential housing are all assumed to be taxed at the same rate. Third, tax rates are not indexed by income or age.

4.1.2 Wage income

Disposable wage income is equal to gross wages minus direct taxes minus social insurance contributions to the public PAYG and private pension systems:

$$WageY(t, age) = WageRate(t, age)Emp(t, age) \quad (4.1.2.1)$$

$$DispWageY(t, age) = WageY(t, age) - DirTaxWageY(t, age) - \\ - ContPubSysWageY(t, age) - ContPenSysWageY(t, age) \quad (4.1.2.2)$$

where $ContPvtPenSysWageY(t, age)$ consists of the sum of contributions to the DB and DC components of the private pension system. Calculations of direct taxes and pension system contributions are described in Sections 6 and 7, respectively. Note that, even though $PvtPenSys$ contributions really represent the acquisition of a financial asset, rather than a current expenditure flow, the System of National Accounts (SNA) nonetheless counts such transactions as a debit in the calculation of disposable income. However, an adjustment is made (see Section 4.1.9) to ensure that the savings associated with such flows are credited to households.

4.1.3 Rental income

Imputed rental income is assumed to be taxed like any other form of income; however, social contributions are assumed to be zero:

$$Rntly(t, age) = r(t)KRes(t, age) \quad (4.1.3.1)$$

$$DispRntly(t, age) = Rntly(t, age) - DirTaxRntly(t, age) \quad (4.1.3.2)$$

Recall, from Section 3.1, that capital returns are already net of depreciation and indirect taxes.

4.1.4 Entrepreneurial income

Profits from capital operated by unincorporated enterprises are treated the same as wages:

$$EntrY(t, age) = r(t)KPUE(t, age) \quad (4.1.4.1)$$

$$DispEntrY(t, age) = EntrY(t, age) - DirTaxEntrY(t, age) \\ - ContrPubPenSysEntrY(t, age) \quad (4.1.4.2) \\ - ContrPvtPenSysEntrY(t, age)$$

4.1.5 Dividend income

The assets held on households' behalf by $PvtPenSys$ and OFI earn dividends. However, in the first case, dividends are not considered by the SNA to be part of household income; rather, they are considered to represent the acquisition of a financial asset. The adjustment described in Section 4.1.9 will add these dividend earnings captured by the private pension system to household income. Unadjusted household disposable income includes only dividends on assets held by OFI :

$$DividY(t, age) = DividDistErningsFirmsKOFI(t, age) \quad (4.1.5.1)$$

The calculation of distributed earnings is given below in Section 5.2.2. Having already been taxed when earned, dividend earnings are not taxed when received by households. Disposable dividend income is thus simply:

$$DispDividY(t, age) = DividY(t, age) \quad (4.1.5.2)$$

4.1.6 Pensions income

Pension income comes from three sources: the public PAYG pension system, the private DC pension system, and the private DB pension system. All three systems provide benefits, which are current transfers in the first case and, while representing sales of capital assets in the second two cases, are nonetheless considered for accounting purposes to represent income. The calculation of pension benefits is described in Section 7. In addition, as described in Section 7.2.2, in any year, some persons will change jobs and a given proportion of these will choose to withdraw their assets from the private pension system rather than rolling them over into new plans. These withdrawals are also treated as income. We assume that withdrawals occur only from the DC pension system. While this point is debatable, however, most countries have in place measures that strongly encourage job-switchers to transfer their DB pension assets into another plan. (The logic behind this is that, on retirement, the benefit entitlement from the private DB pension scheme is calculated on the basis of years of participation and earnings. If we were to allow withdrawal of assets, it would be necessary to "restart the clock" every time assets were withdrawn, or to link benefits with accumulated assets (as in the DC private pension system) rather than earnings.) Combining the two components of the private pension system,

$$PensionY(t, age) = BenPubPenSys(t, age) + BenPvtPenSys(t, age) + WithrwlKPvtPenSys(t, age) \quad (4.1.6.1.)$$

Public pension income is subject to taxation because it is a current transfer. Private pension income and early withdrawals from the DC pension system are not taxed because these represent the sale of capital assets whose returns were taxed at the level of the firm. Disposable pension income is therefore

$$DispPensionY(t, age) = [1 - DirTaxRate(t)]BenPubPenSys(t, age) + BenPvtPenSys(t, age) + WithrwlKPvtPenSys(t, age) \quad (4.1.6.2.)$$

Since private pension system benefits represent the drawing-down of a capital asset, they are included in the adjustment to disposable income described in Section 4.1.9. Early withdrawals (usually in consequence of job change) from the DC private pension system are described in Section 4.2.3; these are also included in the adjustment to disposable income described below.

4.1.7 Total income

Total income of households is equal to the sum over all income sources:

$$TotYHH(t, age) = WageY(t, age) + Rntly(t, age) + EntrY(t, age) + DividY(t, age) + PensionY(t, age) \quad (4.1.7.1.)$$

4.1.8 Disposable income

Disposable income is analogous:

$$DispYHH(t, age) = DispWageY(t, age) + DispRntly(t, age) + DispEntrY(t, age) + DispDividY(t, age) + DispPensionY(t, age) \quad (4.1.8.1.)$$

4.1.9 Adjusted disposable income.

Adjusted disposable income is equal to disposable income

- plus contributions to *PvtPenSys*,
- plus dividends earned on assets held by *PvtPenSys*,
- minus benefits received from *PvtPenSys*.
- minus early withdrawals from the defined contribution private pension system (see Section 4.2.3)

Thus,

$$AdjDispYHH(t, age) = DispYHH(t, age) + ContPvtPenSys(t, age) + DividKPvtPenSys(t, age) - BenPvtPenSys(t, age) - WithrwlKPvtPenSysDC \quad (4.1.9.1.)$$

where the third term represents the sum over DB and DC components of the private pension systems of dividends paid out by firms (given in Sections 7.2.1 and 7.3.1, respectively).

Adjusted disposable income is close to, but not exactly the same as, disposable income plus change in pension wealth. The latter would be equal to

$$\Delta KPvtPenSys(t, age) = ContPvtPenSys(t, age) + DividKPvtPenSys(t, age) - BenPvtPenSys(t, age) - BeqKPvtPenSys(t, age) - WithrwlKPvtPenSysDC(t, age) \quad (4.1.9.2.)$$

where *BeqKPvtPenSys(t, age)* reflects the fact that upon the death of the claimant, accumulated pension assets are paid out to heirs. (From the standpoint of calculating individual wealth, the bequest term is irrelevant, because the individual must die in order to bequeath. In calculating cohort wealth, however, bequests must be taken into account) There is no accounting for inheritance of pension wealth because paying-out is assumed to take the form of cash allocated either to consumption or to the acquisition of non-pension capital assets. Stated differently, there is no explicit modeling of survivors' benefits, which amount to reassigning title to existing pension assets. In conclusion, we could also write

$$AdjDispYHH(t, age) = DispYHH(t, age) + \Delta KPvtPenSys(t, age) + BeqPvtPenSys(t, age) \quad (4.1.9.3.)$$

4.2 Capital transfers

Resources available for household consumption take the form of disposable income and the proceeds of transferring claims to capital assets. In this section, the second of these is elaborated.

4.2.1 Annuitization of assets in old age

Starting at the pension eligibility age, households are assumed to divest themselves of non-pension assets in a way calculated to exhaust assets at age 100. This "annuitization" process -- which we model for simplicity's sake just as a program of asset sales -- is assumed to begin whether households are still in the labor force or not. For $[\bullet]=Res, PUE, OFI$, annuity income is:

$$AssetSalesK[\bullet](t, age) = \frac{K[\bullet](t, age)}{100 - age}, \quad age \geq EligAge \quad (4.2.1.1.)$$

If the propensity to consume out of the proceeds of asset sales is unity, there is no bequest motive; if, for example, the propensity is 0.95, elderly households aim to die with 5 percent of their wealth intact, etc. Note that it is assumed that no assets are sold prior to retirement, apart from the special case of assets received via inheritance and the withdrawal of DC pension assets associated with job change, which we discuss in the next sections.

4.2.2. Bequests/Inheritance

In all asset classes, age-specific bequests are equal to assets times the proportion of persons in the age group dying. For $[\bullet]=Res, PUE, OFI, PvtPenSysDC, PvtPenSysDB$:

$$BeqK[\bullet](t, age) = K[\bullet](t, age) \left[\frac{Deaths(t, age)}{Pop(t-1, age-1)} \right] \quad (4.2.2.1.)$$

Without question, DC pension system assets, like non-pension wealth, belong to the individual and are heritable. As we discuss in Section 8.2, the case of assets of the DB pension system is more debatable. Bequests are received, in the form of inheritance, by the surviving population. For simplicity, we estimate age-specific inheritance simply by dividing total bequests by population age shares. We exclude the population under age 15. Total bequests are

$$BeqK[\bullet](t) = \sum_{age=15}^{MaxAge} BeqK[\bullet](t, age) \quad (4.2.2.2.)$$

and inheritance (for age groups over 15) is

$$InhK[\bullet](t, age) = \left[\frac{Pop(t, age)}{\sum_{age=15}^{MaxAge-1} Pop(t, age)} \right] \sum_{age=15}^{MaxAge} BeqK[\bullet](t, age) \quad (4.2.2.3.)$$

Summing over age groups,

$$InhK[\bullet](t) = \sum_{age=15}^{MaxAge} InhK[\bullet](t, age) \quad (4.2.2.4.)$$

This simplification admittedly exaggerates the number of "backwards" bequests (elderly persons inheriting wealth from middle-aged persons, who are in fact more likely to bequeath assets to their children than to their parents). (One expedient way to solve this problem is to assume that only persons under some age, say 65, but this runs the danger of failing to account for significant spousal bequests. Ultimately, a vector of age-specific share coefficients should be applied to allocate bequests from persons of a given age group over heirs by age group) The assumption is made that, when wealth is inherited, it is converted to cash, some of which is allocated to consumption and the remainder of which is allocated among $KOFI$, $KRes$, and PUE using the same share coefficients applied to household net saving (see Section 8.3). Note, however, that the portion not consumed does not comprise new household savings; it represents the acquisition of a claim formed as the result of past saving.

Under these assumptions, sales of inherited assets are

$$SaleInhK[\bullet](t, age) = InhK[\bullet](t, age) \quad (4.2.2.5.)$$

Consumption out of the proceeds of such sales is described in Section 4.3.4.

4.3 Outlay

4.3.1 Direct taxes.

These are described in Section 6.1.

4.3.2 Social insurance contributions

These are described in Section 7.

4.3.3 Consumption out of income

Average propensities to consume (*AvgPropCons*) out of disposable income streams are exogenous assumptions:

$$\text{ConsWageY}(t, \text{age}) = \text{DispWageY}(t, \text{age}) \text{AvgPropConsWageY}(t, \text{age}) \quad (4.3.3.1.)$$

$$\text{ConsEntrY}(t, \text{age}) = \text{DispEntrY}(t, \text{age}) \text{AvgPropConsEntrY}(t, \text{age}) \quad (4.3.3.2.)$$

$$\text{ConsDividY}(t, \text{age}) = \text{DispDividY}(t, \text{age}) \text{AvgPropConsDividY}(t, \text{age}) \quad (4.3.3.3.)$$

$$\text{ConsBenPubPenSys}(t, \text{age}) = \text{BenPubPenSys}(t, \text{age}) \text{AvgPropConsBenPubPenSys}(t, \text{age}) \quad (4.3.3.4.)$$

$$\text{ConsBenPvtPenSysDC}(t, \text{age}) = \text{BenPvtPenSysDC}(t, \text{age}) \text{AvgPropConsBenPvtPenSysDC}(t, \text{age}) \quad (4.3.3.5.)$$

$$\text{ConsBenPvtPenSysDB}(t, \text{age}) = \text{BenPvtPenSysDB}(t, \text{age}) \text{AvgPropConsBenPvtPenSysDB}(t, \text{age}) \quad (4.3.3.6.)$$

It is assumed that all imputed housing services are consumed:

$$\text{ConsRntlY}(t, \text{age}) = \text{DispRntlY}(t, \text{age}) \quad (4.3.3.7.)$$

4.3.4 Consumption out of the proceeds of asset sales.

It is assumed that consumption out of the proceeds of asset sales takes place in the year of the sale, i.e., households do not hold liquid balances.

4.3.4.1 Consumption out of the proceeds of selling inherited assets

For $[\bullet] = \text{Res}, \text{PUE}, \text{OFI}, \text{PvtPenSysDC}, \text{PvtPenSysDB}$ consumption out of the sales of inherited assets is

$$\text{ConsSaleInhK}[\bullet](t, \text{age}) = \text{SaleInhK}[\bullet](t, \text{age}) \text{ConsShareSaleInhK}[\bullet](t, \text{age}) \quad (4.3.4.1.1.)$$

and the sharing-out of what is not consumed between ΔKOFI , ΔKRes , and ΔKPUE is described in Section 8.3 below. We use a mnemonic corresponding to "consumption share" instead of *AvgPropCons* because average propensity to consume is properly considered with reference to income.

4.3.4.2 Consumption out of retirement annuity income.

Consumption in old age financed by the sale of assets accumulated during working life is treated in the same way. Because private pension system benefits are classified for purposes of the SNA as income, rather than capital transfers, this component has already been described above. For the remaining components $[\bullet] = \text{OFI}, \text{PUE}, \text{Res}$:

$$\text{ConsAssetSalesK}[\bullet](t, \text{age}) = \text{AssetSalesK}[\bullet](t, \text{age}) \text{ConsShareAssetSalesK}[\bullet](t, \text{age}) \quad (4.3.4.2.1.)$$

As mentioned above in discussion private pension system benefits, if there is no bequest motive, the consumption shares are assumed to be unity. However, this assumption can be generalized to allow for bequests. In this case, the complement of the consumption share is simply the proportion of wealth upon retirement which households wish to bequeath.

4.3.4.3 Consumption out of the proceeds of selling DC pension assets withdrawn in consequence of job change.

The final component of consumption is:

$$\text{ConsWithdrwlKPvtPenSysDC}(t, \text{age}) = \text{WithdrwlKPvtPenSysDC}(t, \text{age}) \text{ConsShareWithdrwlKPvtPenSys}(t, \text{age}) \quad (4.3.4.3.1.)$$

Early withdrawals from the private defined-benefit pension system are assumed to be zero.

4.4 Net savings of households

Recapitulating, disposable and adjusted disposable household incomes are

$$\text{DispYHH}(t, \text{age}) = \text{DispWageY}(t, \text{age}) + \text{DispRntlY}(t, \text{age}) + \text{DispEntrY}(t, \text{age}) + \text{DispDividY}(t, \text{age}) + \text{DispPensionY}(t, \text{age}) \quad (4.4.1.)$$

$$\begin{aligned} \text{AdjDispYHH}(t, \text{age}) = & \text{DispYHH}(t, \text{age}) + \text{ContPvtPenSys}(t, \text{age}) + \text{DividPvtPenSys}(t, \text{age}) \\ & - \text{BenPvtPenSys}(t, \text{age}) - \text{WithdrwlKPvtPenSysDC}(t, \text{age}) \end{aligned} \quad (4.4.2.)$$

and total consumption is

$$\begin{aligned} \text{PvtCons}(t, \text{age}) = & \text{ConsDispWageY}(t, \text{age}) + \text{ConsDispRntlY}(t, \text{age}) + \text{ConsDispEntrY}(t, \text{age}) \\ & + \text{ConsDispDividY}(t, \text{age}) + \text{ConsDispPensionY}(t, \text{age}) \\ & + \text{ConsSaleInhKRes}(t, \text{age}) + \text{ConsSaleInhKPUE}(t, \text{age}) \\ & + \text{ConsSaleInhKOFI}(t, \text{age}) + \text{ConsSaleInhKPvtPenSysDC}(t, \text{age}) \\ & + \text{ConsAassetSalesKOFI}(t, \text{age}) + \text{ConsAassetSalesKRes}(t, \text{age}) + \text{ConsAassetSalesPUE}(t, \text{age}) \\ & + \text{ConsWithdrwlKPvtPenSysDC}(t, \text{age}) \end{aligned} \quad (4.4.3.)$$

The first two lines on the right-hand side give consumption out of income (including pension income), the second two lines give consumption financed by the sale of inherited assets, the fifth line gives consumption out of annuity income, and the sixth line covers consumption which occurs when a worker changes jobs and elects to withdraw DC pension assets.

Household net saving is the difference between disposable income and consumption:

$$NetSvngHH(t,age) = DispYHH(t,age) - PvtCons(t,age) \quad (4.4.4)$$

and adjusted net savings includes savings captured by the private pension system:

$$AdjNetSvngHH(t,age) = AdjDispYHH(t,age) - PvtCons(t,age) \quad (4.4.5)$$

or, expressing in terms of unadjusted disposable income and change in pension wealth (see Section 4.1.9),

$$AdjNetSvngHH(t,age) = DispYHH(t,age) + \Delta KPvtPenSys(t,age) + BeqKPvtPenSys(t,age) - PvtCons(t,age) \quad (4.4.6)$$

In performing the consistency check in Section 9 below, we will use this identity in the form

$$NetSvngHH(t,age) = AdjNetSvngHH(t,age) - \Delta KPvtPenSys(t,age) + BeqKPvtPenSys(t,age) \quad (4.4.7)$$

5. Income, outlay, and net savings of firms

Firms operate capital, earn profits and pay out direct taxes and dividends.

5.1 Income

Earnings of firms consist of earnings on capital owned by the three institutional claimants [\bullet] = OFI, PvtPenSysDB, PvtPenSysDC:

$$ErngsFirmsK[\bullet](t,age) = r(t) K[\bullet](t,age) \quad (5.1.1.)$$

$$ErndsFirmsK[\bullet](age) = \sum_{age=0}^{MaxAge} ErngsFirmsK[\bullet](t,age) \quad (5.1.2.)$$

Recall that depreciation and indirect taxes have already been netted out.

5.2 Outlay

5.2.1 Direct taxes

Taxes on profits are described in Section 6.1.

5.2.2 Dividends

Dividend distributions are made out of pre-tax earnings, and the proportion of earnings distributed is independent of the claimant by assumption. For the three claimants [\bullet] = OFI, PvtPenSysDB, PvtPenSysDC:

$$DividDistErngsFirmsK[\bullet](t,age) = DividDistShare(t) ErngsFirmsK[\bullet](t,age) \quad (5.2.2.1.)$$

$$DividDistErngsFirmsK[\bullet](t) = \sum_{age=0}^{MaxAge} DividDistErngsFirmsK[\bullet](t,age) \quad (5.2.2.2)$$

where the share of earnings distributed as dividends is an exogenous variable.

5.3 Net savings of firms

Net savings (retained earnings) of firms are

$$NetSvngErngsFirmsK[\bullet](t,age) = ErngsFirmsK[\bullet](t,age) - DirTaxErngsFirmsK[\bullet](t,age) - DividDistErngsFirmsK[\bullet](t,age) \quad (5.3.1.)$$

The sum over claimants is total net savings of firms:

$$NetSvngFirms(t,age) = \sum_{[\bullet]} NetSvngErngsFirmsK[\bullet](t) \quad (5.3.2.)$$

and the sum over age groups gives total corporate savings:

$$NetSvngFirms(t) = \sum_{age=0}^{MaxAge} NetSvngFirms(t,age) \quad (5.3.3.)$$

6. Income, outlay, and net savings of government

The government sector is rudimentary. Government consumes an exogenous share of GDP, collects taxes and social security contributions and pays social security benefits.

6.1 Income

Government revenues are

$$GovRev(t) = IndTax(t) + DirTax(t) + ContPubPenSys(t) \quad (6.1.1)$$

where

$$IndTax(t) = IndTaxRate(t) GDP(t) \quad (6.1.2)$$

$$\begin{aligned} DirTax(t) = & \sum_{age=15}^{MaxAge} DirTaxWageY(t, age) + \sum_{age=15}^{MaxAge} DirTaxEntrY(t, age) + \\ & + \sum_{age=EligAge}^{MaxAge} DirTaxRntly(t, age) + \\ & + \sum_{age=15}^{MaxAge} DirTaxBenPubPenSys(t, age) + \sum_{age=0}^{MaxAge} \sum_{[\bullet]} DirTaxErngsFirmsK[\bullet] \end{aligned} \quad (6.1.3)$$

where $[\bullet] = OFI, PvtPenSysDB, PvtPenSysDC$ and the direct tax streams are

$$DirTaxWageY(t, age) = DirTaxRate(t, age) WageY(t, age) \quad (6.1.4.)$$

$$DirTaxRntly(t, age) = DirTaxRate(t, age) Rntly(t, age) \quad (6.1.5.)$$

$$DirTaxEntrY(t, age) = DirTaxRate(t, age) EntrY(t, age) \quad (6.1.6.)$$

$$DirTaxBenPubPenSys(t, age) = DirTaxRate(t, age) BenPubPenSys(t, age) \quad (6.1.7.)$$

$$DirTaxErngsFirmsK[\bullet](t, age) = DirTaxRate ErngsFirmsK[\bullet](t, age) \quad (6.1.8.)$$

Contributions to the public pension system are described in Section 7.1.1.

6.2 Outlay

Government expenditure is:

$$GovExp(t) = GovCons(t) + BenPubPenSys(t) \quad (6.2.1.)$$

where government consumption is taken simply as a fixed share of GDP:

$$GovCons(t) = GovConsShare(t) GDP(t) \quad (6.2.2.)$$

and benefits paid out by the public pension system are described in Section 7.1.2.1

6.3 Net savings of government

Government net savings are

$$NetSvngGov(t) = GovRev(t) - GovExp(t) \quad (6.3.1.)$$

Net savings of government are allocated across age groups using shares drawn from the age-distribution of wealth:

7. Pension system

7.1 Public DB PAYG pension system

7.1.1 Income

Contributions to the public pension system out of wages are

$$ContPubPenSysWageY(t, age) = PubPenSysContRate(t) WageY \quad (7.1.1.1.)$$

The social security contribution rate $PubPenSysContRate(t)$ is assumed to be age-independent. No distinction is made between employees' and employers' contributions. Social security contributions out of entrepreneurial income are calculated similarly:

$$ContPubPenSysEntrY(t, age) = PubPenSysContRate(t) EntrY(t, age) \quad (7.1.1.2.)$$

Contribution rates out of wage and entrepreneurial income are assumed to be the same.

Total social security system revenues out of each income stream are

$$ContPubPenSysWageY(t) = \sum_{age=15}^{MaxAge} ContPubPenSysWageY(t, age) \quad (7.1.1.3.)$$

$$ConrPubPenSysEntrY(t) = \sum_{age=15}^{MaxAge} ContPubPenSysEntrY(t, age) \quad (7.1.1.4.)$$

and the system total is:

$$ContPubPenSys(t) = ContPubPenSysWageY(t) + ContPubPenSysEntrY(t) \quad (7.1.1.5.)$$

7.1.2 Outlay

The public pension system is assumed to be a DB system financed on a Pay As You Go (PAYG) basis. Let $BenEntPubPenSys(t, age, RtrmntDuration)$ be the social security benefit entitlement for the average person aged age who retired $RtrmntDuration$ years ago, where we assume that $BenEntPubPenSys(t, age, 0) = 0$. The pension for persons entering retirement is computed according to the formula:

$$BenEntPubPenSys(t, age, 1) = \frac{\left(\sum_{j=1}^{age-14} LabForcePartRate(t-j, age-j) \right) \left(\sum_{k=1}^{Re fPeriod} WageRate(t-k, age-k) \right)}{age-14 \quad Re fPeriod} \quad (7.1.2.1.)$$

$ReplRatePubPenSys(t)$ is a coefficient of proportionality which translates earnings into an initial pension entitlement. This replacement ratio is applied to the average labor force participation rate between age 15 and age (assuming $age \geq EligAge$) times average annual earnings during the $RefPeriod$ years prior to retirement.

Social security system benefits paid out by age group of recipient are equal to the age- and retirement-duration specific entitlement times the number of recipients:

$$BenPubPenSys(t, age) = \sum_{RtrmntDuration=0}^{MaxAge-EligAge+1} BenEntPubPenSys(t, age, RtrmntDuration) Pop(t, age, RtrmntDuration) \quad (7.1.2.2.)$$

where, making the simplifying assumption that once retired, persons stay retired,

$$Pop(t, age, RtrmntDuration) = Pop(t, age)$$

$$\left[\begin{array}{l} LabForcePartRate(t - RtrmntDuration, age - RtrmntDuration) \\ - LabForcePartRate(t - RtrmntDuration + 1, age - RtrmntDuration + 1) \end{array} \right] \quad (7.1.2.3.)$$

for $age = 1, age - RtrmntAge + 1$. System-wide expenditures are equal to the summation over age groups

$$BenPubPenSys(t) = \sum_{age=EligAge}^{MaxAge} BenPubPenSys(t, age) Pop(t, age) \quad (7.1.2.4.)$$

7.1.3 System balance

In a classic PAYG system total contributions are equal to total benefits; there is neither accumulation of a return-generating surplus nor a deficit to be financed out of general government revenue. The default model solution option is one in which the required contribution rate is calculated by setting contributions equal to expenditures. However, there are cases where nominally PAYG public pension systems are currently running large surpluses in an effort to pre-finance the retirement of the baby boom generation. In other cases, deficits in the PAYG pension system are financed by transfers from general tax revenue. To cover such cases, an alternative solution option is to set the contribution rate independent of benefits, in which case the model solves for the implied surplus or deficit.

System balance at time t is calculated as follows:

$$BalPubPenSys(t) = BalPubPenSys(t-1) + (1+r) BalPubPenSys(t-1) + ContPubPenSys(t) - BenPubPenSys(t) \quad (7.1.2.5.)$$

7.2 Private DC pension system

7.2.1 Revenue.

Income of the private DC pension system is comprised of (1) current contributions (zero for persons who have retired), and (2) receipt of dividends. (1) is the sum over contributions out of wage and entrepreneurial income, each consisting of the share of the workforce participating times the proportion of total income contributed:

$$ContPvtPenSysDCWageY(t,age)=[PartSharePvtPenSysDC ContRatePvtPenSysDCWageY]WageY(t,age) \quad (7.2.1.1.)$$

$$ContPvtPenSysDCEntrY(t,age)=[PartSharePvtPenSysDC ContRatePvtPenSysDCEntrY]EntrY(t,age) \quad (7.2.1.2.)$$

Total contributions and dividend earnings are

$$ContPvtPenSysDC(t,age)=ContPvtPenSysDCWageY(t,age)+ContPvtPenSysDCEntrY(t,age) \quad (7.2.1.3.)$$

and

$$DividPvtPenSysDC(t) = \sum_{age=15}^{MaxAge} DividDistErngsFirmsKPvtPenSysDC(t, age) \quad (7.2.1.4.)$$

where the paying-out of dividends was described in Section 5.2.2.

7.2.2. Expenditure

Expenditures of the private DC pension are (1) benefits paid out (zero for persons still in the labor force), (2) pay-out to heirs of the pension assets of system participants who die, and (3) withdrawal of assets by job-switchers who choose not to roll over their pension wealth into another plan. (1) is analogous to the "annuitization" of non-pension capital assets described in Section 4.2.1 above, with the difference that only those who have left the labor force receive pension benefits:

$$BenPvtPenSysDC(t, age) = [1 - LabForcePartRate(t, age)] \frac{KPvtPenSysDC(t, age)}{100 - age} \quad (7.2.2.1.)$$

where $age \geq EligAge$. and 100 is the maximum age to which a person expects to live. (2) was described above in Section 4.2.2. (3) is calculated using an exogenously assumed withdrawal rate reflecting both the number of job-changes and the proportion who choose not to roll over their assets:

$$WithdrwlPvtPenSysDC(t,age)=WithdrwlRatePvtPenSysDC(t,age) KPvtPenSysDC(t,age) \quad (7.2.2.2.)$$

If, for example, 10 percent of system participants change jobs every year and half choose to withdraw their assets, we would have $WithdrwlRatePvtPenSysDC = .05$.

7.3 Private DB pension system

7.3.1 Revenue

This is analogous to the public pension system:

$$ContPvtPenSysDBWageY(t,age)=[PartSharePvtPenSysDB(t,age)ContRatePvtPenSysDB(t,age)]WageY(t,age) \quad (7.3.1.1.)$$

$$ContPvtPenSysDBEntrY(t,age)=[PartSharePvtPenSysDB(t,age) ContRatePvtPenSysDB(t,age)]EntrY(t,age) \quad (7.3.1.2.)$$

and total contributions are the sum over the two sources:

$$ContPvtPenSysDB(t,age)=ContPvtPenSysDBWageY(t,age)+ContPvtPenSysDBEntrY(t,age) \quad (7.3.2.3.)$$

Age-specific dividends are

$$DividKPvtPenSysDB(t,age)= DividDistErngsFirmsKPvtPenSysDB(t,age) \quad (7.3.2.4.)$$

where the paying-out of dividends by firms is described in Section 5.2.2. The total over age groups is

$$DividPvtPenSysBDB(t) = \sum_{age=15}^{MaxAge} DividKPvtPenSysDB(t, age) \quad (7.3.2.5.)$$

7.3.2 Expenditure

The average private DB pension entitlement for a newly-retired person is calculated similarly to the average initial public pension entitlement:

$$BenEntPvtPenSysDB(t, age, 1) = \left[\frac{PartSharePvtPenSysDB \cdot Re \cdot plRatePvtPenSysDB(t)}{\frac{\sum_{j=1}^{age-14} LabForcePartRate(t-j, age-j)}{age-14} \cdot \frac{\sum_{k=1}^{Re \cdot fPeriod} WageRate(t-k, age-k)}{Re \cdot fPeriodPvtPenSysDB}} \right] \quad (7.3.2.1.)$$

Benefits are

$$BenPvtPenSysDB(t, age) = \sum_{RtrmntDuration=0}^{MaxAge-EligAge+1} BenEntPvtPenSysDB(t, age, RtrmntDuration) \cdot Pop(t, age, RtrmntDuration) \quad (7.3.2.2.)$$

We assume that $RtrmntDuration$ is the same for the public PAYG and private DB pension systems. The total benefits paid out are

$$BenPvtPenSysDB(t) = \sum_{age=EligAge}^{MaxAge} BenPvtPenSysDB(t, age) \cdot Pop(t, age) \quad (7.3.2.3.)$$

Bequests of DB pension system assets were described in Section 4.2.2. Recall that withdrawals from the DB pension system associated with job-change were assumed to be zero.

7.4 Total private pension system contributions and benefits

Private pension system totals are

$$ContPvtPenSysWageY(t, age) = ContPvtPenSysDCWageY(t, age) + ContPvtPenSysDBWageY(t, age) \quad (7.3.2.4.)$$

$$ContPvtPenSysEntrY(t, age) = ContPvtPenSysDCEntrY(t, age) + ContPvtPenSysDBEntrY(t, age) \quad (7.3.2.5.)$$

$$DividKPvtPenSys(t, age) = DividKPvtPenSysDC(t, age) + DividKPvtPenSysDB(t, age) \quad (7.3.2.6.)$$

$$BenPvtPenSys(t, age) = BenPvtPenSysDB(t, age) + BenPvtPenSysDC(t, age) \quad (7.3.2.7.)$$

8. The life-cycle dynamics of capital accumulation

Corresponding to each of the types of capital $KPvtPenSysDC$, $KPvtPenSysDB$, $KRes$, $KPUE$ and $KOFI$ is an age-specific capital accumulation identity.

8.1 DC private pension system

Change in age-specific private DC pension wealth is

$$\Delta KPvtPenSysDC(t, age) = ContPvtPenSysDC(t, age) + DividKPvtPenSysDC(t, age) - BenPvtPenSysDC(t, age) - BeqKPvtPenSysDC(t, age) - WithdrwlKPvtPenSysDC(t, age) \quad (8.1.1.)$$

The most important characteristic of the private DC pension system is that there is a fixed relationship between the amount a cohort pays in during its working life and the amount it receives after retirement. For an individual cohort born in year $t = 0$ whose last members die out in year $t = 100$ lifetime pension contributions plus lifetime earnings on pension assets minus lifetime pension benefits received equals bequest of pension wealth. Expressing this differently,

$$\sum_{t=0}^{100} \sum_{age=15}^{100} \Delta KPvtPenSysDC(t, age) = \sum_{t=0}^{100} \sum_{age=15}^{100} \begin{bmatrix} ContPvtPenSysDC(t, age) \\ + DividPvtPenSysDC(t, age) \\ - BenPvtPenSysDC(t, age) \\ - BeqKPvtPenSysDC(t, age) \\ - WithdrwlKPvtPenSysDC(t, age) \end{bmatrix} = 0 \quad (8.1.2.)$$

8.2 DB private pension system

Contributions to the private DC pension system purchase an asset, which is owned by the system participant who made the contribution. Contributions into the private DB pension system, by contrast, purchase a claim on a future pension to be paid by the

firm, which is in turn backed by an asset acquired by the firm. The question of whether assets backing a DB pension scheme belong to system participants or to the firm is a complicated one, and legal regimes differ from country to country. For accounting purposes, however, we treat assets of the DB pension system the same way we treat assets of the DC pension scheme.

$$\Delta K_{pvtPenSysDB}(t, age) = ContPvtPenSysDB(t, age) + DividPvtPenSysDB(t, age) - BenPvtPenSysDB(t, age) - BeqKPvtPenSysDB(t, age) \quad (8.2.1.)$$

For very aged cohorts, unlike in the case of the DC pension scheme, DB pension wealth can be negative. In this case, there is a negative "inheritance" upon death; however, with reasonable parameterization. The number of persons in cohorts characterized by negative DB pension wealth will be small (Let a hypothetical individual contribute 5 percent of annual wages, set equal to 1, from age 20 to 60. Then upon retirement at age 60, assuming an annual rate of return of 6 percent, the individual will have assets equal to 7.7. Now let the individual receive a benefit equal to 0.5 (i.e., the replacement rate is 50 percent) while continuing to earn 6 percent per year on remaining assets. Then at age 100, there are still assets of 1.8 remaining. At the level of the cohort, proportion in the labor force and proportion of workers participating must be taken into account, but as both contributions and benefits are adjusted equally, the same qualitative insight applies).

8.3 Other assets

For $[\bullet] = OFI, PUE, Res$, the age-specific accumulation equations are

$$\begin{aligned} \Delta K[\bullet](t, age) &= K[\bullet]Share(t) \\ &[NetSvngHH(t, age) + NetSvngFirms(t, age) + NetSvngGovt(t, age)] \\ &- AssetSalesK[\bullet](t, age) + K[\bullet]Share(t) \sum_{[\bullet]} AssetSalesK[\bullet] \\ &- BeqK[\bullet](t, age) + InhK[\bullet](t, age) - SaleInhK[\bullet](t, age) + K[\bullet]Share(t) \sum_{[\bullet]} SalaInhK[\bullet] \\ &+ K[\bullet]Share(t)[SaleInhKPvtPenSysDC(t, age) + SaleInhKPvtPenSysDB(t, age)] \end{aligned} \quad (8.3.1.)$$

The components of change are, in order:

- In the first line on the right-hand side of the identity, a share variable $K[\bullet]Share(t, age)$ summing to unity across the three forms of non-pension wealth is used to apportion unadjusted household net savings plus the imputed age-specific savings of firms and government between ΔK_{Res} , ΔK_{PUE} , and ΔK_{OFI} (Allocation shares are not indexed by age for computational simplicity when the model is solved in stochastic mode, however, shares may be age-indexed in non-stochastic model applications)
- The second line on the right-hand side is of relevance only for elderly households. The first term subtracts dissaving in the form of annuitization of assets, as described in Section 4.2.1. The second term, when combined with the consumption from annuity income which is implicit in net household savings in the first line, has the effect of distributing savings from annuity income between the non-pension asset classes.
- The third line on the right-hand side subtracts net bequests (the first two terms) and, analogously to the second line, distributes that portion of inherited wealth not converted into consumption among asset classes. Consumption financed by the sale of inherited assets is not accounted for here because, like consumption from annuity income, it has already been subtracted off in calculating net household savings in the first line.
- The fourth line on the right-hand side distributes inheritance of pension assets between the non-pension asset classes (Note that early withdrawals from the private DC pension system, as well as consumption financed by such withdrawals, are included in net household savings in the first line). Again, associated consumption has already been accounted for when net household savings in the first line are calculated.